# DRAFT

# Content Specifications with Content Mapping for the Summative assessment of the *Common Core State Standards for Mathematics*

## REVIEW DRAFT

## Available for Consortium and Stakeholder Review and Feedback August 29, 2011



**Developed with input from content experts and SMARTER Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee**

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An SBAC-focused version of the Working Group report may be found at
http://www.mathshell.org/papers/pdf/ISDDE_SBAC_Feb11.pdf

# Table of Contents

# INTRODUCTION AND BACKGROUND

**Using This Document:** This version of the SMARTER Balanced Assessment Consortium's work on Content Specifications and Content Mapping is presented as a set of several materials, all with a release date of August 29, 2011. This version, the first of two public releases available for review and feedback, invites commentary from all interested stakeholders in the Consortium's work. Instructions on how to submit comments and feedback can be found in the Resources section of the Consortium's Web site: www.smarterbalanced.org

Pages 1-52 represent the core of this document, and should be read carefully for comment and feedback. Two sets of appendices are intended to provide further elaboration of our work so far. The first set – Appendices A and B – are embedded in this document, as they might be most useful for a reader to have ready at-hand. The second set – Appendix C – is provided as stand-alone resources that provide additional detail to our current developments.

In addition to this document and the addendum of Appendices A-C we are making available two online surveys for stakeholder feedback – one for use by individuals and that will capture responses from a group. We know there is a lot of interest in this release, and anticipate a very large volume of feedback. To ensure that comments and suggestions are received and considered, **we ask readers to be sure to use the online survey** as the vehicle for providing responses.

This document follows an earlier release by the Consortium of a companion document covering specifications for English language arts and literacy. These documents seek comment from Consortium members and other stakeholders. The table below outlines the schedule for the two rounds of public review for the content specifications of mathematics and English language arts/literacy.

## SBAC Content Specifications and Content Mapping

## Development Timelines and Activities

| Review Steps | Date |
|---|---|
| **Internal Review Start: ELA/Literacy**<br>- ELA/Literacy content specifications distributed to specific SBAC work groups for initial review and feedback | 07/05 (Tue) |
| **Internal Review Due: ELA/Literacy**<br>- Emailed to SBAC | 07/15 (Fri) |
| **Technical Advisory Committee (TAC) Review Liaison Review: ELA/Literacy**<br>- Draft submitted to TAC for review, comment, and feedback | 07/27 (Wed) |
| **Webinar: ELA/Literacy (including Evidence Based Design orientation)**<br>- Orientation for SBAC members to Evidence Based Design and walkthrough of draft ELA/Literacy specifications document | 08/08 (Mon) |
| **Release for Review: ELA/Literacy (Round 1)**<br>- ELA/Literacy specifications documents posted on SBAC Web site & emailed to stakeholder groups | 08/09 (Tue) |

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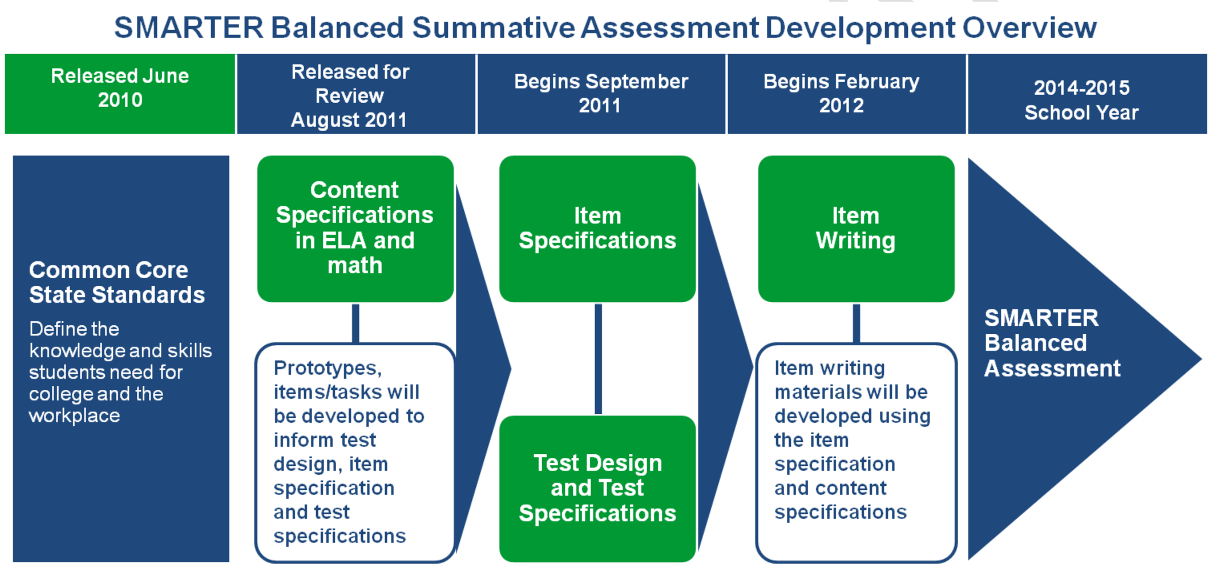
|---|---|
| **Internal Review Start: Mathematics**<br>- Mathematics content specifications distributed to specific SBAC work groups for preliminary review and feedback | 08/10 (Wed) |
| **Technical Advisory Committee (TAC) Review Liaison Review: Mathematics**<br>- Draft submitted to TAC for review, comment, and feedback | 08/10 (Wed) |
| **Internal Review Due: Mathematics**<br>- Emailed to SBAC | 08/15 (Mon) |
| **Release to Item Specifications to Bidders: ELA/Literacy**<br>- Current drafts of ELA/Literacy content specifications posted to OSPI Web site to support Item Specifications RFP process | 08/15 (Mon) |
| **Webinar: Mathematics**<br>- Walkthrough for SBAC members of the draft Mathematics specifications document | 08/29 (Mon) |
| **Release for Review: Mathematics (Round 1)**<br>- Mathematics content specifications posted on SBAC External Site & emailed to stakeholder groups | 08/29 (Mon) |
| **Release of Specifications to Bidders: Mathematics**<br>- Current drafts of Mathematics content specifications posted to OSPI Web site to support Item Specifications RFP process | 08/29 (Mon) |
| **Feedback Surveys Due: ELA/Literacy (Round 1)**<br>- Emailed to SBAC | 08/29 (Mon) |
| **Feedback Surveys Due: Mathematics (Round 1)**<br>- Emailed to SBAC | 09/19 (Mon) |
| **Release for Review: ELA/Literacy (Round 2)**<br>- ELA content specifications posted on SBAC External Site & emailed to stakeholder groups | 09/19 (Mon) |
| **Feedback Surveys Due: ELA/Literacy (Round 2)**<br>- Emailed to SBAC | 09/26 (Mon) |
| **Final Content Specifications and Content Mapping Released: ELA/Literacy**<br>- Final ELA content specifications and content mapping posted to External Web site; email notification sent to member states and partner organizations | 10/03 (Mon) |
| **Release for Review: Mathematics (Round 2)**<br>- Mathematics content specifications posted on SBAC External Site & emailed to stakeholder groups | 10/10 (Mon) |
| **Feedback Surveys Due: Mathematics (Round 2)**<br>- Emailed to SBAC | 10/17 (Mon) |
| **Final Content Specifications and Content Mapping Released: Mathematics**<br>- Final Mathematics content specifications and content mapping posted to External Site; email notification sent to member states and partner organizations | 10/24 (Mon) |

The contents of this document describe the extent of the Consortium's current development to specify critically important claims about student learning that are derived from the Common Core State Standards. When finalized, these claims will serve as the basis for the Consortium's system of summative and interim assessments and its formative assessment support for teachers. Open and transparent decision-making is one of the Consortium's central principles. This draft of the mathematics content specifications is being made available for comment consistent with that principle, and all responses to this work will be considered as it continues to be refined.

**Purpose of the Content Specifications:** The SMARTER Balanced Assessment Consortium is developing a comprehensive assessment system for mathematics and English language arts / literacy— aligned to the Common Core State Standards—with the goal of preparing all students for success in college and the workforce. Developed in partnership with member states, leading researchers, content expert experts, and the authors of the Common Core, content specifications are intended to ensure that the assessment system accurately assesses the full range the standards.

This content specifications of the Common Core mathematics standards provide clear and rigorous prioritized assessment targets that will be used to translate the grade-level Common Core standards into content frameworks along a learning continuum, from which test blueprints and item/task specifications will be established. Assessment evidence at each grade level provides item and task specificity and clarifies the connections between instructional processes and assessment outcomes.




**The Consortium Theory of Action for Assessment Systems:** As stated in the SMARTER Balanced Assessment Consortium's (SBAC) Race to the Top proposal, "the Consortium's Theory of Action calls for full integration of the learning and assessment systems, leading to more informed decision-making and higher-quality instruction, and ultimately to increased numbers of students who are well prepared for college and careers." (p. 31). To that end, SBAC's proposed system features rigorous content standards; common adaptive summative assessments that make use of technology-enhanced item types, and include teacher-developed performance tasks; computer adaptive interim assessments—reflecting learning progressions—that provide mid-course information about what students know and can do; instructionally sensitive formative tools, processes, and practices that can be accessed on-demand; focused ongoing support to teachers through professional development opportunities and exemplary instructional materials; and an online, tailored, reporting and tracking system that allows teachers, administrators, and students to access information about progress towards achieving college- and career-readiness as well as to identify specific strengths and weaknesses along the way. Each of these components serve to support the Consortium's overarching goal: *to ensure that all students leave high school prepared for post-secondary success in college or a career through increased student learning and improved teaching*. Meeting this goal will require the coordination of many elements across the educational system, including but not limited to a quality assessment system that strategically "balances" summative, interim, and formative components (Darling-Hammond & Pecheone, 2010; SBAC, 2010).

The proposed SBAC mathematics assessments and the assessment system are shaped by a set of characteristics shared by the systems of high-achieving nations and states, and include the following principles (Darling-Hammond, 2010):

1) **Assessments are grounded in a thoughtful, standards-based curriculum and are managed as part of an <u>integrated system</u>** of standards, curriculum, assessment, instruction, and teacher development. Curriculum and assessments are organized around a set of learning progressions[1] along multiple dimensions within subject areas. These guide teaching decisions, classroom-based assessment, and external assessment.

2) **Assessments include <u>evidence of student performance</u>** on challenging tasks that evaluate Common Core Standards of 21$^{st}$ century learning. Instruction and assessments seek to teach and evaluate knowledge and skills that generalize and can transfer to higher education and multiple work domains. They emphasize deep knowledge of core concepts and ideas within and across the disciplines, along with analysis, synthesis, problem solving, communication, and critical thinking. This kind of learning and teaching requires a focus on complex performances as well as the testing of specific concepts, facts, and skills.

3) **<u>Teachers are integrally involved</u> in the development and scoring of assessments**. While many assessment components can and will be efficiently and effectively scored with computer assistance, teachers will also be involved in the interim/benchmark, formative, and summative assessment systems so that they deeply understand and can teach the standards.

4) **Assessments are structured to <u>continuously improve teaching and learning.</u>** Assessment *as*, *of*, and *for* learning is designed to develop understanding of what learning standards are, what high-quality work looks like, what growth is occurring, and what is needed for student learning. This includes:

    - <u>Developing assessments around learning progressions</u> that allow teachers to see what students know and can do on multiple dimensions of learning and to strategically support their progress;

[1] Empirically-based learning progressions can visually and verbally articulate a hypothesis, or an anticipated path, of how student learning will typically move toward increased understanding over time with good instruction (Hess, Kurizaki, & Holt, 2009).

- Using computer-based technologies to adapt assessments to student levels to more effectively measure what they know, so that teachers can target instruction more carefully and can evaluate growth over time;
- Creating opportunities for students and teachers to get feedback on student learning throughout the school year, in forms that are actionable for improving success;
- Providing curriculum-embedded assessments that offer models of good curriculum and assessment practice, enhance curriculum equity within and across schools, and allow teachers to see and evaluate student learning in ways that can feed back into instructional and curriculum decisions; and
- Allowing close examination of student work and moderated teacher scoring as sources of ongoing professional development.

5) **Assessment, reporting, and accountability systems provide useful information on multiple measures that is educative for all stakeholders.** Reporting of assessment results is timely, specific, and vivid—offering specific information about areas of performance and examples of student responses along with illustrative benchmarks, so that teachers and students can follow up with targeted instruction. Multiple assessment opportunities (formative and interim/benchmark, as well as summative) offer ongoing information about learning and improvement. Reports to stakeholders beyond the school provide specific data, examples, and illustrations so that administrators and policymakers can more fully understand what students know in order to guide curriculum and professional development decisions.

**Accessibility to Content Standards and Assessments:** In addition to these five principles, SBAC is committed to ensuring that the content standards, summative assessments, teacher-developed performance tasks, and interim assessments adhere to the principles of accessibility for students with disabilities and English Language Learners.[2] It is important to understand that the purpose of accessibility is ***not*** to reduce the rigor of the Common Core State Standards, but rather to avoid the creation of barriers for students who may need to demonstrate their knowledge and skills at the same level of rigor in different ways. Toward this end, each of the claims for the CCSS in Mathematics is briefly clarified in terms of accessibility considerations. Information on what this means for content specifications and mapping will be developed further during the test and item development phases.

[2] Accessibility in assessments refers to moving "beyond merely providing a way for students to participate in assessments. Accessible assessments provide a means for determining whether the knowledge and skills of each student meet standards-based criteria. This is not to say that accessible assessments are designed to measure whatever knowledge and skills a student happens to have. Rather, they measure the same knowledge and skills at the same level as traditional … assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities [or English Language Learners] from what would be measured for peers without disabilities" (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009, p. 2).

Too often, individuals knowledgeable about students with disabilities and English learners are not included at the beginning of the process of thinking about standards and assessments, with the result being that artificial barriers are set up in the definition of the content domain and the specification of how the content maps onto the assessment. These barriers can seriously interfere with the learning of these students, and can prevent them from showing their knowledge and skills via assessments. The focus on "accessibility," as well as the five principles shared by systems of high-achieving nations and states, underlies the Consortium's approach to content mapping and the development of content specifications for the SBAC assessment system.

*Accessibility* is a broad term that covers both instruction (including access to the general education curriculum) and assessment (including summative, interim, and formative assessment tools). *Universal design* is another term that has been used to convey this approach to instruction and assessment (Johnstone, Thompson, Miller, & Thurlow, 2008; Rose, Meyer, & Hitchcock, 2005; Thompson, Thurlow, & Malouf, 2004; Thurlow, Johnstone, & Ketterline Geller, 2008; Thurlow, Johnstone, Thompson, & Case, 2008). The primary concept behind these terms is to move beyond merely providing a way for students to participate in instruction or assessments. Instead, the goals are (a) to ensure that students learn what other students learn, and (b) to determine whether the knowledge and skills of each student meet standards-based criteria.

Several approaches have been developed to meet the two major goals of accessibility and universal design. They include a focus on multiple means of representation, multiple means of expression, and multiple means of engagement for instruction. Elements of universally designed assessments and considerations for item and test review are a focus for developing accessible assessments. Increased attention has been given to computer-based assessments (Thurlow, Lazarus, Albus, & Hodgson, 2010) and the need to establish common protocols for item and test development, such as those described by Mattson and Russell (2010).

For assessments, the goal for all students with disabilities (except those students with significant cognitive disabilities who participate in an alternate assessment based on alternate achievement standards) is to measure the same knowledge and skills at the same level as traditional assessments, be they summative, interim, or formative assessments. Accessibility does not entail measuring different knowledge and skills for students with disabilities from what would be measured for peers without disabilities (Thurlow, Laitusis, Dillon, Cook, Moen, Abedi, & O'Brien, 2009; Thurlow, Quenemoen, Lazarus, Moen, Johnstone, Liu, Christensen, Albus, & Altman, 2008). It does entail understanding the characteristics and needs of students with disabilities and addressing ways to design assessments and provide accommodations to get around the barriers created by their disabilities.

Similarly, the goal for students who are English language learners is to ensure that performance is not impeded by the use of language that creates barriers that are unrelated to the construct being measured. Unnecessary linguistic complexity may affect the accessibility of assessments for all students, particularly for those who are non-native speakers of English (Abedi, in press; Abedi, 2010; Solano-Flores, 2008). For example, several studies have discussed how the performance of ELL students can be confounded during mathematics assessments as a function of unfamiliar cultural referents and unnecessary linguistic complexities (see for example, Abedi, 2010; Abedi & Lord, 2001; Solano-Flores, 2008).

In particular, research has demonstrated that several linguistic features unrelated to mathematics content could slow the reader down, increase the possibility of misinterpretation of mathematics items, and add to the ELL student's cognitive load, thus interfering with understanding the assessment questions and explaining the outcomes of assessments. Indices of language difficulty that may be unrelated to the mathematics content include unfamiliar (or less commonly used) vocabulary, complex grammatical structures, and styles of discourse that include extra material, conditional clauses, abstractions, and passive voice construction (Abedi, 2010a).

A distinction has been made between language that is relevant to the focal construct (mathematics in this case) and language that is irrelevant to the content (construct-irrelevant). SBAC intends to address issues concerning the impact of unnecessary linguistic complexity of mathematics items as a source of construct-irrelevant factor for ELL students, and provide guidelines on how to control for such sources of threat to the reliability and validity of mathematics assessments for these students. Studies on the impact of language factors on the assessment outcomes have also demonstrated that they impact performance of students with learning and reading disabilities. Thus, controlling for such sources of impact will also help students with learning/reading disabilities (Abedi, 2010b).

In addition, ELL students' abilities to communicate could substantially confound their level of proficiency in mathematics, as it is required for many of the mathematical tasks. For example, a major requirement for a successful performance in mathematics as outlined in the CCSSM is a high level of verbal and written communication skills. Each of the four Claims indicates that successful completion of mathematics operations may not be sufficient to claim success in the tasks and that students should also be able to clearly and fluently communicate their reasoning. This could be a major obstacle for ELL students who are highly proficient in mathematical concepts and mathematical operations but not at the level of proficiency in English to provide clear explanation of the operations.

In the case of English learners (EL), ensuring appropriate assessment will require a reliable and valid measure of EL students' level of proficiency in their native language (L1) and in English (L2). In general, if students are not proficient in English but are proficient in L1 and have been instructed in L1, then a native language version of the assessment should be considered, since an English version of the assessment will not provide a reliable and valid measure of students' abilities to read, write, listen, and speak. If students are at the level of proficiency in reading in English to meaningfully participate in an English-only assessment (based, for example, on a screening test or the Title III ELP assessment), then it will be appropriate to provide access in a computer adaptive mode to items that are consistent with their level of English proficiency but measure the same construct as other items in the pool. (See Abedi, et al 2011 for a computer adaptive system based on students' level of English language proficiency.)

As issues of accessibility are being considered, attention first should be given to ensuring that the design of the assessment itself does not create barriers that interfere with students showing what they know and can do in relation to the content standards. Several approaches to doing this were used in the development of alternate assessments based on modified achievement standards and could be brought into regular assessments to meet the needs of all students, not just those with disabilities, once the content is more carefully defined. To determine whether a complex linguistic structure in the assessment is a necessary part of the construct (i.e., construct-relevant), a group of experts (including content and linguistic experts and teachers) should convene at the test development phase and determine all the construct-relevant language in the assessments. This analysis is part of the universal design process.

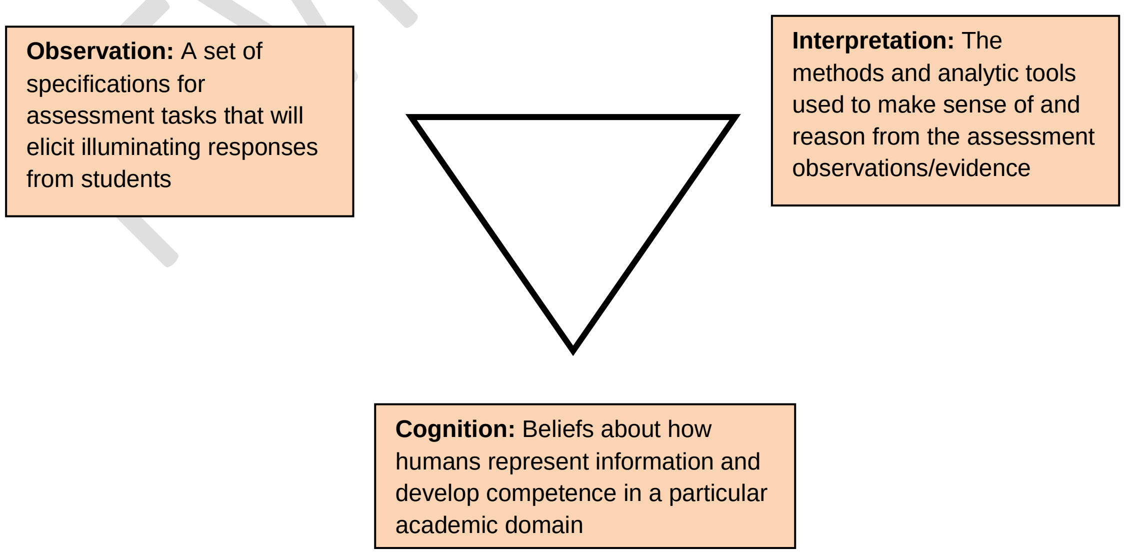
**Further Readings:** Each of the SBAC assessment system principles is interwoven throughout this document in describing the content mapping and content specifications. Readers may want to engage in additional background reading to better understand how the concepts below have influenced the development of the SBAC mathematics assessment design.

- **Principles of evidence-based design (EBD); The Assessment Triangle (see next page); Cognition and transfer; Performances of novices/experts**
  (see Pellegrino, Chudowsky, & Glaser, 2001; Pellegrino, 2002)
- **Enduring understandings, transfer**
  (see Wiggins & McTighe, 2001)
- **Principles of evidence-centered design (ECD) for assessment**
  (see Mislevy, 1993, 1995)
- **Learning progressions/learning progressions frameworks**
  (see Hess, 2008, 2010, 2011; National Assessment Governing Board, 2007; Popham, 2011; Wilson, 2009)
- **Universal Design for Learning (UDL); Increased accessibility of test items**
  (see Abedi, 2010; Bechard, Russell, Camacho, Thurlow, Ketterlin Geller, Godin, McDivitt, Hess, & Cameto, 2009; Hess, McDivitt, & Fincher, 2008).
- **Cognitive rigor, Depth of Knowledge; Deep learning**
  (see Alliance for Excellence in Education, 2011; Hess, Carlock, Jones, & Walkup, 2009; Webb, 1999)
- **Interim assessment; Formative Assessment**
  (see Perie, Marion, & Gong, 2007; Heritage, 2010; Popham, 2011; Wiliam, 2011)
- **Constructing Questions and Tasks for Technology Platforms**
  (see Scalise & Gifford, 2006)

Accommodations then should be identified that will provide access for students who still need assistance getting around the barriers created by their disabilities or their level of English language proficiency after the assessments themselves are as accessible as possible. For example, where it is appropriate, items may be prepared at different levels of linguistic complexity so that students can have the opportunity to respond to the items that are more relevant for them based on their needs, ensuring that the focal constructs are not altered when making assessments more linguistically accessible. Both approaches (designing accessible assessments and identifying appropriate accommodations) require careful definition of the content to be assessed.

Careful definitions of the content are being created by SBAC. These definitions involve identifying the SBAC assessment claims, the rationale for them, what sufficient evidence looks like, and possible reporting categories for each claim. Further explication of these claims provides the basis for ensuring the accessibility of the content – accessibility that does not compromise the intended content for instruction and assessment – as well as accommodations that might be used without changing the content. Sample explications are provided under each of the claims.

**Content Mapping and Content Specifications for Assessment Design:** The Assessment Triangle was first presented by Pellegrino, Chudowsky, and Glaser in *Knowing What Students Know*/KWSK (NRC, 2001.) "[T]he corners of the triangle represent the three key elements underlying any assessment…a model of student ***cognition*** and learning in the domain, a set of beliefs about the kinds of ***observations*** that will provide evidence of students' competencies, and an ***interpretation*** process for making sense of the evidence" (NRC, 2001, p. 44). KWSK uses the heuristic of this 'assessment triangle' to illustrate the fundamental components of evidence-based design (EBD), which articulates the relationships among learning models (Cognition), assessment methods (Observation), and inferences one can draw from the observations made about what students truly know and can do (Interpretation) (Hess, Burdge, & Clayton, 2011).




**The Assessment Triangle** (NRC, 2001, p. 44)

Application of the assessment triangle not only contributes to better test design. The interconnections among Cognition, Observation, and Interpretation can be used to gain insights into student learning. For example, learning progressions offer a coherent starting point for thinking about how students develop competence in an academic domain and how to observe and interpret the learning as it unfolds over time. These hypotheses about typical pathways of learning can be validated, in part, through systematic (empirical) observation methods and analyses of evidence produced in student work samples from a range of assessments.

**Evidence-based Design**: SBAC is committed to using evidence-based design in its development of assessments in the Consortium's system. The SBAC approach is detailed in the following section, but a brief explanation is as follows. In this document, four "Claims" are declared about what students should know and be able to do in the domain of mathematics. Each claim is accompanied by a "Rationale" that provides the basis for establishing the claim as central to mathematics. The Claims and Rationales represent the "cognition" part of the assessment triangle. For each Claim and Rationale there is a section representing the "observation" corner of the triangle. Here, a narrative description lays out the kinds of evidence that would be sufficient to support the claim, which is followed by tables with "Assessment Targets" linked to the Common Core standards. Finally, the "interpretation" corner of the triangle is represented by a section for each claim that lists the "Proposed Reporting Categories" that the assessment would provide.

# Part I – General Considerations for the Use of Items and Tasks to Assess Mathematics Content and Practice

**Assessing Mathematics**

The Common Core State Standards for mathematics require that mathematical content and mathematical practices be *connected* (CCSSM, p. 8). In addition, two of the major design principles of the standards are *focus* and *coherence* (CCSSM, p. 3). Together, these features of the standards have important implications for the design of the SMARTER Balanced assessment system.

**Using Various Types of Items and Tasks to Connect Content and Practice:** There are multiple dimensions to mathematical proficiency, ranging from knowing important mathematical facts and procedures to being able to use that knowledge in the solution of complex problems. The Consortium intends to use a variety of types of assessment items and tasks to assess students' mathematical proficiency. For example, knowledge of how to add fractions, or how to solve two linear equations in two unknowns can be assessed with selected response or completion items. However demonstrating the skills to model a mathematical situation and explain the rationale for the approach depends on deciding what is mathematically important in that situation, representing it with mathematical symbolism, operating on the symbols appropriately, and then interpreting the results in meaningful ways. Assessing this deeper understanding of mathematics will require more complex assessment tasks. As elaborated below, a balanced and meaningful assessment will contain a spectrum of items, ranging from brief items targeting particular concepts or skills to more elaborate items and tasks that demand the application of multiple practices as described in CCSSM. Extended constructed response and performance tasks can afford the opportunity to assess substantial chains of reasoning as the standards require.

**Focus and Coherence:** The principles of focus and coherence on which the CCSSM are based have additional implications for mathematics assessment and instruction. Coherence implies that the standards are more than a mere checklist of disconnected statements; the cluster headings, domains, and other text in the standards all organize the content in ways that highlight the unity of the subject. Focus is a means of allowing more time for students and teachers to master the intricate, challenging, and necessary things in each grade that open the way to a variety of applications even as they form the prerequisite study for future grades' learning. Assessment will reinforce focus and coherence at each grade level by testing for mastery of central and pivotal mathematics rather than covering too many ideas superficially – a key point of the Common Core Standards. This implies a need to set clear and intelligent content priorities in assessment, as we have outlined here. (See Part II).

**Assessing Levels of Expertise:** Assessments should help us make judgments about student ability to (1) demonstrate basic procedural skill, and conceptual understanding, (2) use such knowledge in a context where their work on complex tasks is scaffolded, and (3) use such knowledge in unscaffolded situations that call on substantial chains of reasoning. We will refer to items and tasks as assessing student knowledge and skill at the *novice*, *apprentice*, or *expert* levels, respectively. It is important to emphasize that a student's level of expertise does not refer to the technical level of the mathematics, but rather to the student's ability to call upon and deploy the mathematics they understand in a range of contexts. Indeed, researchers have identified the notion of *adaptive expertise - the ability to use one's knowledge flexibly and adaptively, in new situations* - as the hallmark of proficiency.[3] At each grade level, as students engage with mathematics they progress in their expertise from a "novice" capacity to understand and explain mathematical concepts and procedures to (hopefully) an "expert" capacity to use and apply those concepts and procedures in new and unique situations. This notion of students moving through a novice-to-expert progression is consistent with recent conceptualizations of expertise from cognitive psychology, including van Hiele's "Levels of Geometric Understanding."

Items and tasks at each level make different demands on students and are valuable for particular purposes. A cluster of items and tasks around a complex expert-level scenario will require the student to reason through a number of steps on their own and will assess the student's ability to make strategic decisions on how to tackle and pursue a problem. Offering the same scenario with more scaffolding that guides students along a particular solution path will provide the opportunity to learn more about the student's facility with the mathematical concepts being assessed. Breaking down the task into a sequence of shorter novice-level items and tasks is a useful way, in a particular mathematical context, to assess separate elements of content. SBAC assessments will contain an appropriate balance of items and tasks at these three levels of expertise.

Items and tasks assessing student understanding of core concepts and procedures in mathematics should draw upon grade-level standards to ensure student mastery of this content; more complex scenarios will ask students to engage in more open-ended framing, modeling, and solving of problems. For these more complex, non-routine combinations of items and tasks the technical demand should focus on concepts and skills that students have thoroughly absorbed and connected together – often those first met in earlier grades.

**Judicious Coverage of the Standards:** CCSSM describes a body of mathematical content and practices that students are to learn. Thus, an assessment consistent with CCSSM must be faithful to that description of the mathematics, taken as a whole, and not simply to individual standards. Such assessments can be constructed by a) beginning with the selection of complex items and

[3] See, e.g., Hatano, G. and K. Inagaki (1986). Two courses of expertise. Child development and education in Japan: 262–272.

tasks that are consistent with the vision of expertise presented in the CCSSM; b) augmenting them with additional apprentice-level items/tasks so as to *even out* the representation of mathematical practices, while simultaneously *concentrating the collection as a whole more deeply* on high-priority content; and c) completing the assessment with a set of novice items/tasks that collectively reflects the focus and coherence of the standards, again concentrating on high-priority content.

**Strategic Uses of Technology:** Wherever possible, computer-adaptive testing (CAT) is a desirable and efficient mechanism for testing and scoring. A range of selected response items (multiple-choice, drag-and-drop, and other categorization tasks) can be scored easily by computer, as can short constructed response items that require a straightforward answer, and many kinds of longer constructed response items. Much of a balanced assessment can be conducted using these tools. Technology also offers many powerful opportunities for working in mathematics, particularly the ability to rapidly and accurately perform large numbers of calculations and to produce sophisticated visualizations. Appropriate use of such technology in assessment can improve balance in assessment by making higher-level thinking and understanding less expensive and more realistic (e.g. choosing the best statistical measures and representations for analyzing a data set with 1000 records, as opposed to selecting the median in a list of a dozen whole numbers).

"Technology enhanced" CAT tasks can also be designed to provide evidence for mathematical practices that could not be obtained from short/selected answer tasks, and can encourage classroom use of authentic mathematical computing tools (spreadsheets, interactive geometry, computer algebra, graphers) for classroom instruction.

At the same time, for much school-level mathematics, paper and pencil remains the natural medium for working mathematically, as it allows for diverse representations such as quick sketches of diagrams or graphs, and for mathematical expressions and tables to be rapidly created and freely mixed. Doing similar exploratory work on a computer would require the time-consuming use of multiple specialized tools, which were often designed for producing polished presentations or setting up large-scale computations rather than as a "scratchpad" for mathematical thinking. Sometimes only the end result of this work needs to be evaluated in the assessment – and it can be entered as an answer for computer scoring. At other times, the work itself is important to assess. For example, evaluating students' capacities to develop "multiple solution paths" and to "choose appropriate tools" requires an open-ended response format. Consequently, a useful blend of methods for working out problems and capturing students' mathematical ideas will be important to achieve.

# Part II – Content Specifications: Mapping Assessment Targets to Standards

### Claims and Evidence for CCSS Mathematics Assessment

**Defining Assessment Claims and Sufficient Evidence:** The theory of action articulated by the Consortium illustrates the vision for an assessment system that will lead to inferences that ensure that all students are well-prepared for college and careers after high school. “Inference is reasoning from what one knows and what one observes, to explanations, conclusions, or predictions. One attempts to establish the weight and coverage of evidence in what is observed” (Mislevy, 1995, p 2). *Claims* are the broad statements of the assessment system’s learning outcomes, each of which requires *evidence* that articulates the types of data/observations that will support interpretations of competence towards achievement of the claims. A first purpose of this document is to identify the critical and relevant claims that will “identify the set of knowledge and skills that is important to measure for the task at hand” (Pellegrino, Chudowsky, and Glaser, 2001), which in this case are the learning outcomes for the CCSS for mathematics.

**Four Major Claims for the SMARTER Balanced Assessment Consortium’s assessments of the *Common Core State Standards for Mathematics***

***Claim #1* - Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

***Claim #2* - Students can frame and solve a range of complex problems in pure and applied mathematics.**

***Claim #3* - Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

***Claim #4* - Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

Relevant and sufficient evidence needs to be collected in order to support each claim. As discussed in the previous section, this can be accomplished using a variety of assessment items and tasks applied in different contexts. Data collection for the SBAC mathematics assessments is designed to be used to measure and make interpretations about within- and across-year student progress. The sufficient evidence section includes, for each claim, a brief analysis of the assessment issues to be addressed to ensure accessibility to the assessment for all students. **Each**

**claim is accompanied with a description of the sufficient relevant evidence from which to draw inferences or conclusions about learning.**

**Assessment Targets:** Tables that display assessment targets follow the description of sufficient evidence to support each claim. These summative assessment targets (evidence) at each grade level represent the prioritized content for assessment.

Suggested classroom-based interim and formative assessment targets are being developed and will be provided in subsequent versions of this document. The classroom-based interim and formative assessment targets will represent smaller learning chunks that teachers can use to monitor ongoing progress in the classroom of critical learning and/or content standards.

> NOTE
>
> The assessment targets after each claim in this document are shown for three grade levels only: Grade 4, Grade 8, and Grade 11.
>
> Assessment targets will be built for each grade level, grades 3-11. However, the Consortium wants to have this review document available to the field for review and feedback before expanding the current targets to other grade levels.

Each of the Assessment Target tables

- **Indicates proposed prioritized content for the summative assessment**: The assessment targets link the Common Core standards for mathematics to the kinds of items and tasks to which students will be expected to respond.
- **Shows how one or more (or parts) of the Common Core standards addresses the target**: Each target is mapped back to the CC standards. Item developers will refer to specific Common Core standards when writing passage-specific items.
- **Indentifies the intended Depth of Knowledge level for assessment targets and test items/tasks:** The likely depth-of-knowledge level (DOK) for each is provided. (The schema used for the DOK designations is provided in Appendix B of this document.)
- Illustrates how assessment targets relate to an across-grade progression of learning embodied in the prioritization of content shown in Appendix A

The annotated graphic below uses an excerpt from the assessment targets for Claim #1, Grade 4, mathematical concepts and procedures, showing the features of the Assessment Target tables, and how to read/interpret them.

## Grade 4 SUMMATIVE ASSESSMENT TARGETS

## Providing Evidence Supporting Claim #1

### Mathematics Claim #1

**Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

**Content for this claim will be sampled from grade 4 clusters consistent with a content prioritization as described in Appendix A. This content can be assessed using a combination of selected response and short constructed response items, but may also be evaluated at a deeper level within long constructed response items and performance tasks. Sampling of Claim 1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims 2, 3, and 4[4].**

**Operations & Algebraic Thinking**

1. **USE OPERATIONS:** Solve multistep problems using operations, including problems requiring interpretation of remainders (e.g., with fractional answers);Using whole numbers and the four operations, find the unknown when given an equation (including using letters for unknown quantities); **Standards: 4.OA-2, 4.OA- 3** (DOK 1, DOK 2)

2. **EXPRESS RELATIONSHIPS:** Use mathematical symbols to interpret, explain, or represent relationships, including multiplicative comparisons or rules for patterns **Standards: 4.OA-1, 4.OA-2, 4.OA-5, 4.NBT-2** (DOK 1, DOK 2)

3. **FACTORS & MULTIPLES:** Determine factors and multiples of whole numbers (1-100); Identify prime and composite numbers **Standards: 4.OA-4** (DOK 1)

4. **ANALYZE PATTERNS:** Generate geometric and numeric patterns when given a rule (using addition, subtraction, multiplication, and/or division); analyze and extend patterns **Standards: 4.OA-4, 4.OA-5** (DOK 2)

Grade and Claim # shown

Text of Claim is provided

General conditions, emphasis, or assessment constraints on what is presented to students are shown here

Required items / targets with range of standards to draw from

Targets are mapped to standards from CCSS

Depth of knowledge level(s) intended for each target are shown



[4] For example, if under claim #3, a reasoning task in a given year involves rewriting expressions involving radicals or rational exponents, then it is unlikely that this will also be assessed under claim #1 in a selected response item. However, under target 1, claim #1, students might be asked to evaluate expressions with integer exponents on the same assessment when not assessed with a longer task under another claim.

**Proposed Reporting Categories:** The summative assessment for mathematics will generate an overall "mathematics" score to meet accountability reporting requirements. In addition, a score will be generated for each of the four claims. There are likely to be a sufficient number of score points for Claims #1 (Concepts and Procedures) to support the reporting of performance at a more detailed level, if not at the individual student, perhaps at aggregated levels of classrooms or schools. The table below summarizes the current formulation of reporting categories that could be derived from the assessment targets.

**Summary of Proposed Score Reporting Categories**

<table>
<tr><td colspan="4">Total Score for Mathematics</td></tr>
<tr><td>Concepts and Procedures Score</td><td rowspan="8">Problem Solving Score</td><td rowspan="8">Communicating Reasoning</td><td rowspan="8">Modeling and Data Analysis</td></tr>
<tr><td><u>Gr 3 C&P Sub-scores</u><br>Operations & Algebraic Thinking<br>Number/Ops – Fractions<br>Measurement & Data</td></tr>
<tr><td><u>Gr 4 C&P Sub-scores</u><br>Operations & Algebraic Thinking<br>Number/Ops – Base 10<br>Number/Ops – Fractions<br>Measurement & Data</td></tr>
<tr><td><u>Gr 5 C&P Sub-scores</u><br>Operations & Algebraic Thinking<br>Number/Ops – Fractions<br>Measurement & Data<br>Geometry</td></tr>
<tr><td><u>Gr 6 C&P Sub-scores</u><br>Ratio & Proportion<br>Number System<br>Expressions & Equations<br>Geometry / Statistics & Probability</td></tr>
<tr><td><u>Gr 7 C&P Sub-scores</u><br>Ratio & Proportion / Number System<br>Expressions & Equations<br>Geometry<br>Statistics & Probability</td></tr>
<tr><td><u>Gr 8 C&P Sub-scores</u><br>Expressions & Equations<br>Functions<br>Geometry</td></tr>
<tr><td><u>High School C&P Sub-scores</u><br>Number & Quantity<br>Algebra<br>Functions</td></tr>
</table>

# Part III – Claims, Rationales, Evidence and Assessment Targets

| *Mathematics Claim #1* |
|---|
| **Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.** |

## *Rationale for Claim #1*

This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together, and why mathematical procedures work in the way that they do. This relates to the structural nature of mathematics:

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. (Practice 7, CCSSM)
>
> They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. (Practice 7, CCSSM)
>
> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x – 1)(x + 1), (x – 1)(x2 + x + 1), and (x – 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (Practice 8, CCSM)

Assessments should include items/tasks that test the precision with which students are able to carry out procedures, describe concepts and communicate results.

> Mathematically proficient students … state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. (Practice 6, CCSSM)

Items/tasks should also assess how well students are able to use appropriate tools strategically.

> Students are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

Many individual content standards in CCSSM set an expectation that students can *explain why* given procedures work.

> One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a + b)(x + y) and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding (a + b + c)(x + y). Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness. (CCSSM, p.4).

Finally, throughout the K-6 standards in CCSSM there are also individual content standards that set expectations for fluency in computation (e.g., fluent multiplication and division within the times tables in Grade 3). Such standards are culminations of progressions of learning, often spanning several grades, that involve conceptual understanding, thoughtful practice, and extra support where necessary. Technology may offer the promise of assessing fluency more thoughtfully than has been done in the past. This, too, is part of 'measuring the full range of the standards.



## *What sufficient evidence looks like for Claim #1*

Evidence on each student's progress along the progressions of mathematical content is the focus of attention in assessing this claim.

**Essential properties of items and tasks that assess this claim**

Tasks assessing this category will include short items – including multiple-choice, other selected response, and short constructed response items – that focus on a particular skill or concept. They will also include items that require students to translate between representations of concepts (words, diagrams, symbols) and items that require the identification of structure. The content priorities for each grade are discussed further in Appendix A.

**Short items** that probe individual concepts or skills have a clear role in assessing this claim, particularly for assessing progress on the recently learned content described in the content standards for each grade[5]. Assessment through short items provides evidence of achievement in conceptual understanding and procedural skills at the "novice level" that is the starting point for the process of gradually building the connections that integrate those "tools" into each student's developing "mathematical expertise".

- **Selected response items** can probe conceptual understanding, particularly when the distractors are chosen to embody common misconceptions. In designing such items, it is essential to try to make sure that students do not obtain correct answers because of "test taking skills" rather than understanding of the mathematical content.
- **Constructed response** items assess mathematical thinking directly; short items of this kind provide direct evidence on students' mastery of standard procedures.
- **Highly scaffolded tasks**, where the student is guided through a series of short steps set in a common problem context, offer another approach to the design of short constructed response items.
- **Items** of any of these kinds have the practical advantage that they can be administered and scored using technology (e.g. computer adaptive testing).

**Extended learned procedures**. This claim also requires the assessment of more extended examples of procedural skills that students may be expected to have learned and practiced. These will include the following task types**:**

- **Standard applications of mathematics** are assessed through exercises where students use important concepts and skills to tackle problem situations that should be in the learned part of the curriculum.
- **Translation tasks**, where students are asked to represent concepts in different ways and translate between representations (words, numbers, tables, graphs, symbolic algebra).



**Use of "Scenarios."** CCSSM stresses the need for balance in assessing individual standards and performance asserting that "…both are assessable using mathematical tasks of sufficient richness."

The Consortium will develop "scenarios" that combine different types of items and tasks requiring students to address compound problems that draw upon content and procedural understanding as well as the mathematical practices. These scenarios embedded in Claim 1 are designed to provide evidence on the other claims and will be an important source of evidence on

[5] (As already explained, this is because the strategic demands of rich tasks adds to the challenges from the content to make them more difficult than short exercises on separate concepts and skills that students have been taught and practiced.)

students' technical skills.

**Accessibility and Claim #1:** This claim clarifies the importance of conceptual understanding and procedural knowledge underlying the important core content in CCSSM. The CCSS refer to the ability to carry out procedures, describe concepts, communicate results, use appropriate tools strategically, and explain why specific procedures make sense. Neither the claim itself nor the CCSS explicitly address the challenges that some students with disabilities face in the area of mathematical calculations. Because of the importance of building skills in computation in early schooling, the explication of the content may be different in early school grades compared to later school grades. Providing assistive technologies such as an abacus or calculator may not be considered appropriate up through about grade 4. At some point during intermediate grades, the use of these tools is considered an appropriate avenue of access to allow students to demonstrate that they are able to "calculate accurately and efficiently.

It is also important to address access to mathematics via decoding text and written expression. The uses of alternative means of access and expression are ones used by successful individuals (Reitz, 2011) to demonstrate high levels of success, and thus are an appropriate avenue of access to the content for students with disabilities in the areas of reading decoding and fluency as well as for those with blindness or visual impairments. Likewise, allowing students alternative ways to express their understanding of mathematics content is important. Students who are unable to explain mathematical processes via writing or computer entry might instead provide their explanation via speech to text technology (or a scribe) or via manipulation of physical objects.

A major aspect of all the claims, including Claim 1, is communication, especially students' ability to *explain why* given procedures work. To allow access to English learners who are at a lower proficiency in writing and speaking, it will be important to provide multiple opportunities for ELL students to communicate their understanding through performance tasks or other approaches where opportunity for multiple domain input can be provided. Furthermore, when a major performance difference exists between tasks such as expanding and explaining, provide more opportunity for the student to express their views through other methods such as the use of native language.

**About the "Summative Assessment Targets" that follow…**

The following pages identify summative assessment targets that describe the evidence that will be used to support Claim #1. Summative assessment targets do not replace the Common Core standards; rather, they reference specific standards at each grade level that test developers will use to guide item and task development and collectively serve the purpose of providing a consistent sampling plan for assessment within and across grades.

The targets that are provided are for grades 4, 8, and 11, serving as elementary, middle, and high school examples of the targets that the Consortium will develop for grades 3-11. The summative assessment targets at each grade level represent the prioritized content for assessment. Suggested classroom-based interim and formative assessment targets are being developed, representing smaller learning chunks that teachers can use to monitor ongoing progress in the classroom of critical learning and/or content standards.

Each assessment target is accompanied by the related standard(s) in the CCSS from which it is drawn, and by the intended cognitive rigor/depth-of-knowledge (DOK) required by the assessment target. (The schema for DOK used here appears in Appendix B.)

## Grade 4 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #1- Concepts and Procedures

### Mathematics Claim #1

**Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

**Content for this claim will be sampled from grade 4 clusters consistent with a content prioritization as described in Appendix A. This content can be assessed using a combination of selected response and short constructed response items, but may also be evaluated at a deeper level within long constructed response items and performance tasks. Sampling of Claim 1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims 2, 3, and 4[6].**

**Operations & Algebraic Thinking**

5. **USE OPERATIONS:** Solve multistep problems using operations, including problems requiring interpretation of remainders (e.g., with fractional answers);Using whole numbers and the four operations, find the unknown when given an equation (including using letters for unknown quantities); **Standards: 4.OA-2, 4.OA- 3** (DOK 1, DOK 2)
6. **EXPRESS RELATIONSHIPS:** Use mathematical symbols to interpret, explain, or represent relationships, including multiplicative comparisons or rules for patterns **Standards: 4.OA-1, 4.OA-2, 4.OA-5, 4.NBT-2** (DOK 1, DOK 2)
7. **FACTORS & MULTIPLES:** Determine factors and multiples of whole numbers (1-100); Identify prime and composite numbers **Standards: 4.OA-4** (DOK 1)
8. **ANALYZE PATTERNS:** Generate geometric and numeric patterns when given a rule (using addition, subtraction, multiplication, and/or division); analyze and extend patterns **Standards: 4.OA-4, 4.OA-5** (DOK 2)

**Number & Operations Base Ten**

9. **PLACE VALUE:** Compare multi-digit whole numbers; Express multi-digit whole numbers using expanded form; Explain the meaning of place value using multi-digit whole numbers. **Standards: 4.NBT-1, 4.NBT-2** (DOK 1, DOK 2)
10. **ROUNDING:** Apply place value understanding to round multi-digit whole numbers **Standards: 4.NBT-3** (DOK 1)
11. **FLUENCY:** Add and subtract multi-digit whole numbers; multiply and divide whole numbers (multiply whole numbers of up to four digits by a one-digit whole number, multiply two two-digit numbers; divide whole numbers of up to four digits by 1-digit divisors) **Standards: 4.NBT-4, 4.NBT-5, 4.NBT-6** (DOK 1)

**Number & Operations Fractions**

12. **EQUIVALENCE:** Generate or represent equivalent fractions (concretely, graphically, and symbolically); explain equivalent fractions in the form a/b; express mixed numbers as equivalent fractions; express fractions with denominator of 10 or 100 as equivalent decimals **Standards: 4.NF-1, 4.NF-3; 4.NF-5, 4.NF-6** (DOK 1, DOK 2)
13. **ORDERING:** Compare fractions with different numerators and different denominators; compare two decimals to the hundredths place **Standards: 4.NF-2, 4.NF-7** (DOK 1, DOK 2)
14. **OPERATIONS WITH FRACTIONS**: Add and subtract fractions with the same denominator; add and subtract mixed numbers with the same denominator; multiply a fraction by a whole number; **Standards: 4.NF-3, 4.NF-4** (DOK 1, DOK 2)

**Continued…**



[6] For example, if under claim #2, a problem solving task in a given year involves addition of fractions, then it is unlikely that addition of fractions will also be assessed under claim #1 in a selected response item. However, under target 10, claim #1, students might be asked to subtract or multiply fractions on the same assessment when not assessed with a longer task under another claim.

**…continued**

**Measurement & Data**

15. **MEASUREMENT UNITS:** Identify relative sizes of units within the same system (customary or metric units); generate or complete conversion tables to express equivalent measurements or measurement estimates **Standards: 4.MD-1** (DOK 1, DOK 2)
16. **APPLY MEASUREMENT CONCEPTS:** Find area and perimeter of rectangles, including using formulas **Standards: 4.MD-3** (DOK 1)

**Geometry**

17. **LINES and ANGLES:** Use attributes of geometric figures (parallel and perpendicular lines; right, acute, obtuse angles) to compare or classify two-dimensional figures; locate or draw lines and line segments (including lines of symmetry in figures); draw and measure angles in whole number degrees **Standards: 4.MD-5, 4.MD-6; 4.G-1, 4.G-2, 4.G-3** (DOK 1, DOK 2)

| Grade 8 SUMMATIVE ASSESSMENT TARGETS<br>Providing Evidence Supporting Claim #1- Concepts and Procedures |
|---|
| **Mathematics Claim #1**<br>**Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.** |
| **Content for this claim will be sampled from grade 8 clusters consistent with a content prioritization as described in Appendix A. This content can be assessed using a combination of selected response and short constructed response items, but may also be evaluated at a deeper level within long constructed response items and performance tasks. Sampling of Claim 1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims 2, 3, and 4[7].** |
| **The Number System**<br>**1. RATIONAL-IRRATIONAL NUMBERS:** Distinguish rational numbers from irrational numbers, by converting a repeating decimal to a rational number, showing that decimal expansions of rational numbers eventually repeat, identifying rational number approximations of irrational numbers (e.g., locating on a number line diagram), or comparing size/magnitude or estimating the value of an expression with irrational numbers **Standards: 8.NS-1, 8.NS-2** (DOK 1, DOK 2)<br><br>**Expressions and Equations**<br>**2. INTEGER EXPONENTS**: Use properties of integer exponents to generate equivalent numerical expressions, evaluate expressions, and estimate or compare very large and very small quantities **Standards: 8. EE.1, 8. EE.3** (DOK 1, DOK 2)<br>**3. RADICALS:** Use square and cube root symbols to represent solutions to equations**;** evaluate square roots (small perfect square roots) and cube roots (small perfect cube roots) **Standards: 8. EE.2** (DOK 1)<br>**4. SCIENTIFIC NOTATION:** Perform operations with numbers expressed in scientific notation; select and use appropriate units expressed in scientific notation to solve problems **Standards: 8. EE.4** (DOK 1, DOK 2)<br>**5. PROPORTIONAL RELATIONSHIPS:** Graph proportional relationships; compare proportional relationships when represented in different ways; interpret proportional relationships (slope, using lines, and linear equations, graphs, unit rates, similar triangles) **Standards: 8. EE.5, EE.6** (DOK 1, DOK 2)<br>**6. LINEAR EQUATIONS:** Analyze and solve linear equations in one variable and pairs of simultaneous linear equations in two variables; estimate solutions by graphing equations **Standards: 8.EE.7, 8.EE.8** (DOK 1, DOK 2)<br><br>**Functions**<br>**7. EVALUATE & COMPARE FUNCTIONS:** Use the properties of functions represented algebraically (equations), graphically (ordered pairs), numerically in tables (input-output), or using verbal rules/descriptions to interpret and explain linear and nonlinear functions and to compare or interpret a function in terms of the situation it is modeling (e.g., rate of change) **Standards: 8.F.1, 8.F.2, 8.F-3, 8.F-5** (DOK 2)<br>**8. MODEL RELATIONSHIPS:** Construct functions to model a linear relationship; determine or interpret rate of change when given graphs, numeric tables, or verbal descriptions; construct graphs to illustrate linear and nonlinear relationships **Standards: 8.F.4, 8.F.5** (DOK 2)<br><br>**Continued…** |

[7] For example, if under claim 2, a problem solving task involves finding volumes of spheres, then it is unlikely that volume of spheres will also be assessed under claim #1 in a selected response item. However, under target #11, claim #1, students might be asked to find the volume of cylinders or cones.

**…continued**

**Geometry**

**9.** **CONGRUENCE & SIMILARITY:** Use properties of rotations, reflections, and translations to explain or verify congruence or similarity of two-dimensional figures (e.g., describe or test a sequence that exhibits congruence or similarity) **Standards: 8.G.1, 8.G-2, 8.G-4** (DOK 2, DOK 3)

**10.** **PYTHAGOREAN THEOREM:** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles or the distance between two points in a coordinate system; use the converse of the Pythagorean Theorem to classify triangles by angle measure. **Standards: 8.G-6, 8.G.7. 8.G-8** (DOK 2)

**11.** **VOLUME of CYLINDERS, CONES, & SPHERES:** Apply formulas for finding the volume of cylinders, cones and spheres **Standards: 8.G.9** (DOK 2)

**Statistics and Probability**

**12.** **BIVARIATE DATA:** Construct or interpret scatter plots for bivariate measurement data (e.g., describe patterns, such as clustering, outliers, linear or nonlinear association); construct or interpret two-way tables summarizing data on two categorical variables **Standards: 8.SP-1, 8.SP-4** (DOK 2)

## Grade 11 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #1- Concepts and Procedures

### Mathematics Claim #1
### Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

**Content for this claim will be sampled from high school clusters consistent with a content prioritization as described in Appendix A. This content can be assessed using a combination of selected response and short constructed response items, but may also be evaluated at a deeper level within long constructed response items and performance tasks. Sampling of Claim 1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims 2, 3, and 4[8].**

**Number and Quantity**

1. **EXPRESSIONS and EXPONENTS:** Use properties of exponents to rewrite expressions involving radicals and rational exponents or to evaluate expressions with integer and rational exponents. **Standards: N-RN-1, N-RN-2** (DOK 1)
2. **NUMBER SYSTEMS:** Apply understanding of rational and irrational numbers to calculate or explain sums and products; apply the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers **Standards: N-RN-3, N-CN-1, N-NC-2** (DOK 1)

**Algebra**

3. **EQUIVALENT EXPRESSIONS**: Recognize or generate equivalent forms of an expression, including rational, polynomial and exponential expressions; use equivalent forms of an expression to construct a rough graph or to reveal and explain specific properties (e.g., the maximum value for the function defined by a quadratic expression). **Standards: A.SSE-2, A.SSE-3, A.APR-2, A.APR-3, A.APR-6, F.IF-8** (DOK 1, DOK 2)
4. **SOLVE EQUATIONS & INEQUALITIES**: Solve linear and quadratic equations in one variable and linear inequalities in one variable; explain each step in solving a simple equation; rearrange formulas to highlight a quantity of interest; explain a graph as a solution set for an equation in two variables; determine the expression for the inverse of a function. **Standards: A.CED-4, A.REI-1, A.REI-2, A.REI-3, A.REI-4, A.REI.10, F-BF.4** (DOK 1, DOK 2)
5. **SOLVE SYSTEMS OF EQUATIONS & INEQUALITIES**: Solve systems of equations and inequalities in two variables, including those represented by a quadratic and linear pair of equations or inequalities. **Standards: A.REI-6, A.REI-7, A.REI-11, A.REI-12** (DOK 1, DOK 2)

**Functions**

6. **RECOGNIZE FUNCTIONAL RELATIONSHIPS:** Distinguish between relationships that represent functions and those that do not; identify the domain and range of a function; recognize a sequence as a function. **Standards: F.IF-1, F.IF-3** (DOK 1, DOK 2)
7. **GRAPHICAL REPRESENTATIONS OF FUNCTIONS:** Sketch graphs based on given key features; identify the corresponding function for a given graph or vice versa, including trigonometric functions; use a graph to evaluate a function for a given value of either variable; create graphs of linear, square root, cube root, piecewise-defined, exponential, and logarithmic functions; relate the domain of a function to its graph; explain the effects of a given transformation on the graph of a function (e.g., the effect on the graph of f(x) of replacing f(x) by f(x) + k). **Standards: A.CED-2, A.APR-3, F.IF-1, F.IF-2, F.IF-4, F.IF-5, F.IF-6, F.IF-7, F.BF-3, F-TF.5** (DOK 1, DOK 2)
8. **COMPARE FUNCTIONS:** Compare properties of two functions represented in different ways; Distinguish between situations that can be modeled with linear and exponential functions; given two or more functions represented graphically or in tables, identify which will exceed the others as the independent variable approaches infinity. **Standards: F.IF-9, LE-1, F.LE-3** (DOK 1, DOK 2)
9. **REPRESENT RELATIONSHIPS BETWEEN QUANTITIES:** Create equations to represent relationships between quantities; represent relationships between two quantities using function notation; write arithmetic and geometric sequences recursively and with an explicit formula, represent exponential models in logarithmic form. **Standards: A.CED-1, A.CED-2, F.IF-2, F.BF-1, F.BF-2, F.LE-2, F.LE-4** (DOK 1, DOK 2)

**Continued…**

[8] For example, if under claim #3, a reasoning task in a given year involves rewriting expressions involving radicals or rational exponents, then it is unlikely that this will also be assessed under claim #1 in a selected response item. However, under target 1, claim #1, students might be asked to evaluate expressions with integer exponents on the same assessment when not assessed with a longer task under another claim.

**…continued**

**Geometry**

10. **TRIGONOMETRIC RATIOS:** Apply understanding of trigonometric ratios and the Pythagorean Theorem to solve problems with right triangles **Standards: G-SRT-8** (DOK 2)
11. **GEOMETRIC RELATIONSHIPS:** Explain or use geometric concepts to show relationships: between sine and cosine of complementary angles; between similarity and side ratios in right triangles; among similarity, congruence, and geometric transformations, with given two figures **Standards: G-SRT-2, G-SRT-5, G-SRT-6, G-SRT-7** (DOK 2)

**Statistics and Probability**

12. **USE and INTERPRET DATA:** Use data presented in a variety of representations (e.g., dot plots, histograms, box plots, two-way frequency tables, scatter plots, data from a sample surveys) to summarize, estimate, or make predictions based on data for a single count or measurement variable or two categorical quantitative variables; or to interpret slope (rate of change) and intercept of a linear model in the context of data **Standards: S-ID-1, S-ID-4, S-ID-5, S-ID-6, S-ID-7, S-IC-4** (DOK 2)
13. **ANALYZE DATA:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread of two data sets; interpret differences in shape, center, and spread; use mean and standard deviation to fit it to a normal distribution **Standards: S-ID-2, S-ID-3, S-ID-4** (DOK 2)

| Mathematics Claim #2 |
| --- |
| **Students can frame and solve a range of complex problems in pure and applied mathematics.** |

Assessment items and tasks focused on this claim include well-posed problems in pure mathematics and problems set in context. *Problems* are presented as items and tasks for which a solution path is not immediately obvious. They require students to construct their own solution pathway, rather than to follow a provided one. Such problems will therefore be unstructured and students will need to select appropriate conceptual and physical tools to use.

## *Rationale for Claim #2*

At the heart of doing mathematics is making sense of problems and persevering in solving them. This claim addresses the core of mathematical expertise – the set of competences that students can use beyond the classroom.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. (Practice 1, CCSSM)

In addition to procedural competence, problem solving is the major action students apply to mathematical content and concepts. Proficiency at problem solving requires students to choose to use concepts and procedures from across the content domains and check their work using alternative methods. As problem solving skills develop, student understanding of and access to mathematical concepts becomes more deeply established.

For example, older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can approach and solve a problem by drawing upon different mathematical characteristics, such as: correspondences among equations, verbal descriptions of mathematical properties, tables graphs and diagrams of important features and relationships, graphical representations of data, and regularity or irregularity of trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. (Practice 1, CCSSM)

Development of the capacity to solve problems also corresponds to the development of important meta-cognitive skills such as oversight of a problem-solving process while attending to the details. Mathematically proficient students continually evaluate the reasonableness of their intermediate results, and can step back for an overview and shift perspective. (Practice 7, Practice 8, CCSM)

Problem solving also requires students to identify and select the tools that are necessary to apply to the problem. The development of this capacity – to frame a problem in terms of the steps that need to be completed and to review the appropriateness of various tools that are available – are critical to further learning in mathematics, and generalize to real-life situations. This includes both mathematical tools and physical ones:

Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. (Practice 5, CCSSM)

## *What sufficient evidence looks like for Claim #2*

Problem solving is at the heart of doing mathematics[9]. Validity *for this purpose* is the key challenge in task design. Tasks whose primary purpose is to assess this claim[10] carry 20% of the total score in the overall assessment of mathematics.

**Essential properties of tasks that assess this claim**

The rationale for this claim makes it clear that evidence for it depends on “expert tasks” that:

- present non-routine[11] problems where a substantial part of the challenge is in deciding what to do, and which mathematical tools to use;
- involve chains of autonomous[12] reasoning, taking a successful student at least 5 to 10 minutes, depending on the age of student and complexity of the task, including explanation of assumptions and conclusions as well as reliable representational and

[9] See, e.g., Halmos, P. (1980). The heart of mathematics. *American Mathematical Monthly*, 87, 519-524

[10] They will also contribute to Claim 2.

[11] As noted earlier, by “non-routine” we mean that the student will not have been taught a closely similar problem, so will not expect to *remember* a solution path but to have to *adapt* or *extend* their earlier knowledge to find one.

[12] By “autonomous” we mean that the student responds to a single prompt, without further guidance within the task.

procedural skills.

It is recognized that such tasks will be new to many students. *For some of these tasks*, therefore, some scaffolding into a sequence of subtasks may be used to facilitate entry and assess student progress towards expertise. The degree of scaffolding for individual students may also be determined as part of the adaptability of the computer-administered test. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Tasks for Claim #2 will normally be designed so that a successful student will complete them in 10-20 minutes; some longer tasks may also contribute evidence for this claim.

**The scoring rubric** for each task should reflect the values set out for this claim, giving substantial weight to the choice of appropriate methods of tackling the task, to reliable skills in carrying it through, and to explanations of what has been found. The content involved may also gain credit under Claim #1.

The set of Claim #2 tasks in a test will sample the content domains. (Many such tasks involve more than one domain)  Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades.

**Some types of tasks that would be suitable**

The following types of task, when well designed and developed through piloting, naturally produce evidence on the aspects of a student's performance relevant to this Claim. We note that such tasks, once released, become the basis of rich classroom conversations about mathematical content and connections – another primary goal of CCSSM.

- **Problems in pure mathematics** Well-posed problems within mathematics that where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results
- **Design problems** have much the same properties but within a design context from the real, or a fantasy, world.
- **Planning problems**   Planning problems involve the coordinated analysis of time, space, cost – and people. They are design tasks with a time dimension added. Well-posed problems assess the student's ability to make the connections needed between different parts of mathematics.

This is not a complete list (were such a thing possible); other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #2. Illustrative examples of each type are given with the Attainment Targets in Part II, and in the sample test in Part III.

**Accessibility and Claim #2:**  This claim about mathematical problem solving focuses on the student's ability to make sense of problems, construct pathways to solving them, persevering in solving them, and the selection and use of appropriate tools. This claim includes student use of appropriate tools for solving mathematical problems, which for some students may extend to tools that provide full access to the item or task and to the development of reasonable solutions. For example, students who are blind and use Braille or assistive technology, such as text readers to access written materials, may demonstrate their modeling of physical objects with geometric shapes using alternate formats. Students who have physical disabilities that preclude movement of arms and hands should not be precluded from demonstrating with assistive technology their use of tools for constructing shapes. As with Claim 1, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to show what they know and can do in relation to framing and solving complex mathematical problems.

With respect to English learners, the expectation for verbal explanations of problems will be more achievable if formative materials and interim assessments provide illustrative examples of the communication required for this claim, so that ELL students have a better understanding of what they are required to do. In addition, formative tools can help teachers teach ELL students ways to communicate their ideas through simple language structures in different language modalities such as speaking and writing. Finally, attention to English proficiency in shaping the delivery of items (e.g. native language or linguistically modified, where appropriate) and the expectations for scoring will be important.

**About the "Summative Assessment Targets" that follow…**

The following pages identify summative assessment targets that describe the evidence that will be used to support Claim #2. Summative assessment targets do not replace the Common Core standards; rather, they reference specific standards at each grade level that test developers will use to guide item and task development and collectively serve the purpose of providing a consistent sampling plan for assessment within and across grades.

The targets that are provided are for grades 4, 8, and 11, serving as elementary, middle, and high school examples of the targets that the Consortium will develop for grades 3-11. The summative assessment targets at each grade level represent the prioritized content for assessment. Suggested classroom-based interim and formative assessment targets are being developed, representing smaller learning chunks that teachers can use to monitor ongoing progress in the classroom of critical learning and/or content standards.

Each assessment target is accompanied by the related standard(s) in the CCSS from which it is drawn, and by the intended cognitive rigor/depth-of-knowledge (DOK) required by the assessment target. (The schema for DOK used here appears in Appendix B.)

## Grade 4 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #2- Problem Solving

### Mathematics Claim #2

**Students can frame and solve a range of complex problems in pure and applied mathematics.**

**Content for this claim will be drawn from grade 4 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Operations & Algebraic Thinking**

1. **SOLVE PROBLEMS using WHOLE NUMBERS:** Given a scenario or context, plan and apply a strategy for arriving at a solution to a multistep problem using whole numbers, which may include: generating equations, diagrams, or other representations to support the solution, identifying patterns, and using the four operations **Standards: 4.OA-3, 4.OA-4, 4.OA- 5** (DOK 3)

**Number & Operations Fractions**

2. **SOLVE PROBLEMS using FRACTIONS:** Given a scenario or context, plan and apply a strategy for arriving at a solution to a multistep problem using addition, subtraction, or multiplication with fractions, mixed numbers, and/or decimal notation for fractions, which may include: generating equations or visual representations (e.g., diagrams, number lines, diagrams, area models) to support the solution **Standards: 4.NF-3; 4.NF-4** (DOK 3)

**Measurement & Data**

3. **SOLVE PROBLEMS using DATA:** Given a scenario, context or data, plan and apply a strategy for arriving at a solution to a multistep problem involving distances, time intervals, liquid volumes, masses of objects, or money. Solutions may require: using units within the same system (customary or metric units), conversions of units in the same system, and use of equations, diagram, data tables, or other representations to support the solution **Standards: 4.MD-2, 4.MD-3** (DOK 3)
4. **SOLVE PROBLEMS using MEASUREMENT:** Given a scenario or context involving area and/or perimeter, plan and apply a strategy for arriving at a solution to a multistep problem (e.g., what happens to area or perimeter when the dimensions of the figure are changed). Solutions may require: recognizing patterns, conversions of units in the same system, and strategies to calculate area or perimeter **Standards: 4.MD-3** (DOK 3)

## Grade 8 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #2- Problem Solving

### Mathematics Claim #2

**Students can frame and solve a range of complex problems in pure and applied mathematics.**

**Content for this claim will be drawn from grade 8 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Expressions and Equations**

1. **SOLVE PROBLEMS using PROPORTIONAL RELATIONSHIPS:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using proportional relationships, which may include using slope, equations, graphs, unit rates, or similar triangles and generating diagrams or other representations to support the solution **Standards: 8. EE.5, EE.6** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)
2. **SOLVE PROBLEMS using LINEAR EQUATIONS:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using linear equations in one variable or pairs of simultaneous linear equations in two variables. Solutions may include graphing equations and generating diagrams or other representations to support the solution **Standards: 8.EE.7, 8.EE.8** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Functions**

3. **SOLVE PROBLEMS using FUNCTIONS:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using the properties of functions, which may include representing linear or nonlinear relationships algebraically (equations), graphically (ordered pairs), numerically in tables (input-output), or using verbal rules/descriptions to support the solution **Standards: 8.F.1, 8.F.2, 8.F-3, 8.F.4, 8.F-5** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Geometry**

4. **SOLVE PROBLEMS using PYTHAGOREAN THEOREM:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using the Pythagorean Theorem (e.g., decomposing a square in two different ways; analyzing polygons) **Standards: 8.G.7. 8.G-8** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)
5. **SOLVE PROBLEMS using VOLUME of CYLINDERS, CONES, & SPHERES:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using formulas for finding the volume of cylinders, cones and spheres **Standards: 8.G.9** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

## Grade 11 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #2- Problem Solving

**Mathematics Claim #2**

**Students can frame and solve a range of complex problems in pure and applied mathematics.**

**Content for this claim will be drawn from high school clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Number and Quantity**

1. **SOLVE PROBLEMS:** Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem finding sums and/or products using rational and irrational numbers; or require making decisions about use of units in formulas, choosing and interpreting scale and origin for data displays, or choosing a level of accuracy appropriate for the limitations on measurement when reporting quantities **Standards: N-RN-3, N-NQ-1, N-NQ-3** (DOK 2)

**Algebra**

2. **SOLVE PROBLEMS using EQUATIONS**: Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem using equations in one or two variables, including problems that require solving systems of equations or generalizing a formula. **Standards: A-CED.1, A.CED-2, A.REI-11, F.BF-1, F.BF-2** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Functions**

3. **SOLVE PROBLEMS by INTERPRETING KEY FEATURES**: Given a scenario or real-world context, plan and apply a strategy for arriving at a solution to a multistep problem that results from examining functions or their graphs and doing one or more of the following: interpreting key features of the function(s) in the context of the problem; interpreting restrictions to the domain based on the function itself or based on the context of the problem; interpreting parts of an expression. **Standards: A-SSE.1, F.IF-4, F.IF-5, F.IF-6, F.IF-8, F.LE-5** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Geometry**

4. **SOLVE SPATIAL PROBLEMS:** Given a scenario or real-world design problem (e.g., interpreting a schematic drawing, estimating the amount of material needed for a sloping roof, designing a tile pattern, describing/visualizing geometric shapes using algebraic formulas), plan and apply a strategy for arriving at a solution to a multistep problem, which may include: trigonometric ratios, concepts of similarity or congruence, or the application of the Pythagorean Theorem **Standards: G-SRT-2, G-SRT-5, G-SRT-6, G-SRT-8, G-GPR-5** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

| *Mathematics Claim #3* |
| --- |
| **Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.** |

This claim refers to a fundamental aspect of mathematics, the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim and ask students to provide, for example, a justification or counter-example.

## *Rationale for Claim #3*

Rigor is about precision in argument: first avoiding making false statements, then saying more precisely what you assume, and providing the sequence of deductions you make on this basis. Assessments should also include tasks that examine a student's ability to analyze a provided explanation, identify flaws, and correct them.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (Practice 3, CCSSM)

Assessment should include tasks that test a student's proficiency in using concepts and definitions in their explanations:

> Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. (Practice 6, CCSSM)

## *What sufficient evidence looks like for Claim #3*

This aspect of performance is at the heart of mathematical reasoning. Validity *for this purpose* is the key challenge in task design. Tasks whose primary purpose is to assess this claim carry 20% of the total score in the overall assessment of mathematics.

**Essential properties of tasks that assess this claim**

The rationale for this claim makes it clear that evidence for it depends on "expert tasks" that:

- present a situation in which either propositions are given or in which students are encouraged to make their own conjectures;
- ask students to test propositions or conjectures with specific examples;
- ask students to construct, autonomously[13], chains of reasoning that will justify or refute the propositions or conjectures; these chains should typically take a successful student 10 minutes[14] or more;
- *for a minority of these tasks*, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes

Tasks for Claim #3 will normally be designed so that a successful student will complete them in 10-20 minutes; some longer tasks may also contribute evidence for this claim.

**The scoring rubric** for each task should reflect the values set out for this claim, giving substantial weight to the quality and precision of the reasoning in several of the following:

- an explanation of the assumptions made;
- the construction of conjectures that appear plausible, where appropriate;
- the quality of the examples that the student constructs in order to evaluate the proposition or conjecture;
- the reasoning that the student uses to describe flaws or gaps in an argument;

[13] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.
[14] Times will be somewhat shorter for younger students, but still giving them time to think and explain.

- the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- the precision with which the student describes the domain of validity of the proposition or conjecture.

The set of Claim #3 tasks in a test will sample the content domains. Some such tasks may involve more than one domain. Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades.

**Some types of task that would be suitable**

The following types of task, when well-designed and developed through piloting, naturally produce evidence on the aspects of a student's performance relevant to this Claim. Examples of each are given in Part II, with an analysis of what each assesses.

- **Proof and justification tasks.** These begin with a proposition and the task is to provide a reasoned argument why the proposition is or is not true. In other tasks, students may be asked to characterize the domain for which the proposition is true.
- **Critiquing tasks.** Some flawed 'student' reasoning is presented and the task is to correct and improve it.
- **Mathematical investigations.** Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to go on and prove one of their conjectures. This kind of task benefits from a longer time scale (from about 20 minutes to many hours)

This is not a complete list; other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #3. . Illustrative examples of each type are given with the Assessment Targets in Part II, and in the sample items and tasks in Part III.

**Accessibility and Claim #3:** Successful performance under Claim 3 requires a high level of linguistic proficiency. Many students with disabilities have difficulty with written expression, whether via putting pencil to paper or fingers to computer. The claim does not suggest that correct spelling or punctuation is a critical part of the construction of a viable argument, nor does it suggest that the argument has to be in words. Thus, for those students whose disabilities create barriers to development of text for demonstrating reasoning and formation of an argument, it is appropriate to model an argument via symbols, geometric shapes, or calculator or computer graphic programs. As for Claims 1 and 2, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to construct viable arguments.

The extensive communication skills anticipated by this claim may also be challenging for many ELL students who nonetheless have mastered the content. Thus it will be important to provide multiple opportunities to ELL students for explaining their ideas through different methods and at different levels of linguistic complexity. Based on the data on ELL students' level of proficiency in L1 and L2, it will be useful to provide opportunities as appropriate for bilingual explanations of the outcomes. Furthermore, students' engagement in critique and debate should not be limited to oral or written words, but can be demonstrated through diagrams, tables, and structured mathematical responses where students provide examples or counter-examples of additional problems.

**About the "Summative Assessment Targets" that follow…**

The following pages identify summative assessment targets that describe the evidence that will be used to support Claim #3. Summative assessment targets do not replace the Common Core standards; rather, they reference specific standards at each grade level that test developers will use to guide item and task development and collectively serve the purpose of providing a consistent sampling plan for assessment within and across grades.

The targets that are provided are for grades 4, 8, and 11, serving as elementary, middle, and high school examples of the targets that the Consortium will develop for grades 3-11. The summative assessment targets at each grade level represent the prioritized content for assessment. Suggested classroom-based interim and formative assessment targets are being developed, representing smaller learning chunks that teachers can use to monitor ongoing progress in the classroom of critical learning and/or content standards.

Each assessment target is accompanied by the related standard(s) in the CCSS from which it is drawn, and by the intended cognitive rigor/depth-of-knowledge (DOK) required by the assessment target. (The schema for DOK used here appears in Appendix B.)

## Grade 4 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #3- Communicating Reasoning

### Mathematics Claim #3

### Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

**Content for this claim will be drawn from grade 4 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Operations & Algebraic Thinking**

1. **REASONING with EQUATIONS & REPRESENTATIONS:** Given one or two possible solutions to a multistep problem using whole numbers (e.g., two students solved this problem in different ways…are they both correct?), use equations, drawings, calculations, or other representations to determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 4.OA-3, 4.OA-4, 4.OA- 5** (DOK 3)

**Number & Operations Base Ten**

2. **REASONING with PLACE VALUE:** Use place value understanding, properties of operations, and representations (equations, rectangular arrays, area models, etc.) to confirm and justify an estimate or explain the reasonableness of a calculation, estimation, or proposed solution (applying multiplication or division) **Standards: 4.NBT-5, 4.NBT-6** (DOK 3)

**Number & Operations Fractions**

3. **REASONING with FRACTIONS & MIXED NUMBERS:** Given one or two possible solutions to a multistep problem using addition, subtraction, or multiplication of fractions or mixed numbers, use equations, calculations, and/or other representations to determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 4.NF-3; 4.NF-4** (DOK 3)

**Measurement & Data**

4. **REASONING with MEASUREMENT & DATA:** Given a context (data, a scenario, possible approach or solution) for a problem involving distances, time intervals, liquid volumes, masses of objects, or money, use equations, calculations, and representations to determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 4.MD-2, 4.MD-3** (DOK 3)

**Geometry**

5. **REASONING with GEOMETRY:** Use attributes and properties of geometric figures (parallel and perpendicular lines; right, acute, obtuse angles) to explain classifications of two-dimensional figures, verify lines of symmetry in geometric figures; or use angle measurements in arriving at and explaining a solution **Standards: 4.MD-5, 4.MD-6; 4.G-1, 4.G-2, 4.G-3** (DOK 2, DOK 3)

**Grade 8 SUMMATIVE ASSESSMENT TARGETS**
**Providing Evidence Supporting Claim #3- Communicating Reasoning**

**Mathematics Claim #3**

**Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

**Content for this claim will be drawn from grade 8 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**The Number System**

1. **REASONING with RATIONAL-IRRATIONAL NUMBERS:** Use understanding of rational and irrational numbers and properties of operations to confirm and justify an estimate or explain the reasonableness of a calculation, estimation, or proposed solution, using equations, diagrams, calculations, or other representations to determine (a) the accuracy of the solution, and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 8.NS-1, 8.NS-2** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Expressions and Equations**

2. **REASONING with PROPORTIONAL RELATIONSHIPS:** Given one or two possible solutions to a multistep problem using proportional relationships (e.g., slope, linear equations, graphs, unit rates, similar triangles) use graphing, equations, diagrams, or other representations to determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 8. EE.5, EE.6** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

3. **REASONING with LINEAR EQUATIONS:** Given one or two possible solutions to a multistep problem using linear equations in one variable or pairs of simultaneous linear equations in two variables, use graphing, equations, diagrams, or other representations to determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 8.EE.7, 8.EE.8** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Functions**

4. **REASONING with FUNCTIONS:** Given one or two possible solutions to a multistep problem using relationships represented algebraically (equations), graphically (ordered pairs), numerically in tables (input-output), or using verbal rules/descriptions, determine (a) the accuracy of the solution(s), and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 8.F.1, 8.F.2, 8.F-3, 8.F.4, 8.F-5** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Geometry**

5. **REASONING with CONGRUENCE & SMILARTY:** Verify congruence or similarity of figures in order to provide justification for the correct outcome or flaws in the reasoning (e.g., test a sequence that exhibits congruence or similarity and explain why it does or does not support the result; establish facts about angle sums and exterior angles of triangles) **Standards: 8.G.1, 8.G-2, 8.G-4, 8.G-5** (DOK 3)
6. **REASONING with PYTHAGOREAN THEOREM:** Provide an explanation/support for a solution using the Pythagorean Theorem (e.g., explaining how to find distances or lengths/heights of real-world structures; analyzing polygons; finding distances between points in a coordinate system) **Standards: 8.G-7, 8.G-8** (DOK 3)

**Statistics and Probability**

7. **REASONING with BIVARIATE DATA:** Given a context (data set(s), a scenario, possible solution, equation of a linear model) representing bivariate measurement, determine (a) the accuracy of the interpretation or solution, and (b) the reasoning behind the strategy used in order to provide justification for the best approach, the correct outcome, or flaws in the reasoning **Standards: 8.SP-1, 8.Sp-3, 8.SP-4** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Grade 11 SUMMATIVE ASSESSMENT TARGETS**
**Providing Evidence Supporting Claim #3- Communicating Reasoning**

**Mathematics Claim #3**

**Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

**Content for this claim will be drawn from high school clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Number and Quantity**

1. **REASONING with QUANTITIES:** Explain the reasoning for decisions made when solving multistep problems, including: use of units in formulas, use of scale and origin for data displays, defining appropriate quantities for descriptive modeling, or choosing a level of accuracy appropriate for the limitations on measurement when reporting quantities **Standards: N-NQ-1, N-NQ-2, N-NQ-3** (DOK 2)

**Algebra**

2. **REASONING with POLYNOMIALS & RATIONAL EXPRESSIONS**: Provide sufficient reasoning to show that polynomials are closed under addition, subtraction, and multiplication or identify flaws in counterexamples presented; prove polynomial identities or explain why a given proof is insufficient, flawed, or sufficient. **Standards: A.APR-1, A.APR-4** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

3. **REASONING with EXPRESSIONS & EQUATIONS**: Derive formulas from given information; provide reasoning to support different methods for solving a given equation; critique methods for equation solving by identifying flawed or insufficient reasoning; provide reasoning that demonstrates an understanding of methods used for solving equations or systems of equations. **Standards: A.SSE-4, A.REI-1, A.REI-4, A.REI-5** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Functions**

4. **REASONING with REPRESENTATIONS OF FUNCTIONS:** Provide sufficient reasoning to explain the growth patterns of linear and exponential functions; provide sufficient reasoning to justify the point of intersection of the graphs of two functions as the solution to a system (extend this to include systems with no solution and those with infinitely many solutions); explain why a function's graph has a particular shape by constructing an argument around a non-graphical representation of the function; given a table of values for a function, construct a graphical representation to justify changes in the function's value for a specified interval of the independent variable. **Standards: A-REI.11; F.IF-7, F.LE-1** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

**Geometry**

5. **REASONING with PROOFS:** Explain a proof of the Pythagorean Theorem and its converse; or explain proofs of theorems about lines, angles, triangles, or parallelograms **Standards: G-SRT-4, G-CO-9, G-CO-10, G-CO-11** (DOK 3)

**Statistics and Probability**

6. **REASONING with DATA:** Provide sufficient reasoning to show that data related to a given a context or scenario will support inferences made or conclusions drawn; or identify flawed or insufficient reasoning based on data analyses **Standards: S-ID-2, S-ID-3, S-ID-4, S-ID-5, S-ID-6, S-ID-7** (DOK 2, DOK 3 if providing justification for or articulating the reasonableness of the solution, using evidence)

*Mathematics Claim #4*

**Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

## *Rationale for Claim #4*

Modeling is the bridge across the “school math”/”real world” divide that has been missing from many mathematics curricula and assessments[15]. It is the twin of *mathematical literacy*, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

> Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decision-making. (p.72, CCSSM)
>
> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (Practice 4; CCSSM)

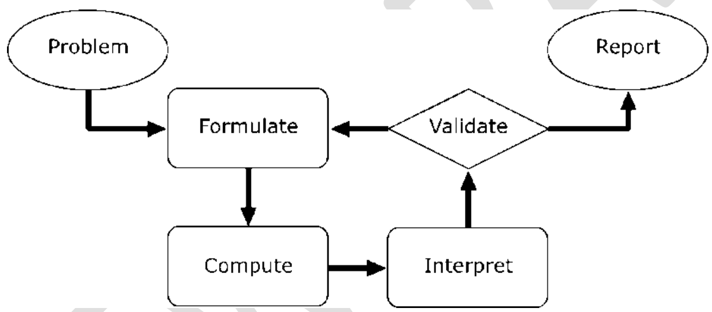
In the real world, problems do not come neatly ‘packaged’. Real world problems are complex, and often contain insufficient or superfluous data. Assessment tasks will involve *formulating* a problem that is tractable using mathematics. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand, or estimate data that are missing. (Such tasks are therefore distinct from the problem solving tasks described in claim 2 that are well-formulated). Students will identify variables in a situation, and construct relationships between these. When students have formulated the problem, they then tackle it,

[15] In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. The mathematics they do use (in planning, personal accounting, design, thinking about political issues etc.) they often do not see as mathematics.

often in a decontextualized form, before interpreting their results and checking them for reasonableness.

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. (Practice 2; CCSSM)

Finally, students interpret, validate and report their solutions through the successive phases of the modeling cycle, illustrated in the following diagram from CCSSM.




Assessment tasks will also test whether students are able to use technology in this process.

> When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

## *What sufficient evidence looks like for Claim #4*

**Essential properties of tasks that assess this claim**

The rationale for this claim makes it clear that evidence for it depends primarily on "expert tasks" that:

- present non-routine[16] problems from the real world where the solution involves some or all of the phases of the modeling cycle;
- for some tasks, a substantial part of the challenge is in formulating an approach: deciding what to do, and which mathematical tools to use;
- involve substantial chains of autonomous[17] reasoning, taking a successful student at least 10 minutes[18], including explanation of assumptions, interpretations, evaluations, and conclusions as well as reliable representational and procedural skills;
- *for a minority of these tasks,* scaffolding into a sequence of subtasks will be used to facilitate entry and assess student progress towards expertise; even for such “apprentice tasks” part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes

Tasks for Claim #4 will normally be designed so that a successful student will complete them in 10-20 minutes; some longer tasks may also contribute evidence for this claim.

**The scoring rubric** for each task should reflect the values set out for this claim, giving substantial weight to the choice of appropriate methods of attacking the task, to reliable skills in carrying it through, and to explanations of what has been found. The content involved may also gain credit under Claim #1.

The set of Claim #4 tasks in a test will sample the content domains. (Many such tasks involve more than one domain) Because of the high strategic demand that substantial non-routine tasks present, the technical demand will be lower – normally met by content first taught in earlier grades.

**Some types of task that would be suitable**

The following types of task, when well-designed and developed through piloting, naturally produce evidence on the aspects of a student’s performance relevant to this Claim. Examples of each are given in Part II, with an analysis of what it assesses.

- **Making decisions from data.** These tasks require students to select from a data source, analyze the data and draw reasonable conclusions from it. This will often result in an *evaluation or recommendation*.
- **Making reasoned estimates.** These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they *do not know*.
- **Planning and design tasks.** Students recognize that this is a problem situation that arises in life and work. Well-posed planning tasks involving the coordinated analysis of time,

[16] As noted earlier, by “non-routine” we mean that the student will not have been taught a closely similar problem, so will not expect to *remember* a solution path but to have to *adapt* or *extend* their earlier knowledge to find one.
[17] By “autonomous” we mean that the student responds to a single prompt, without further guidance within the task.
[18] Times will be somewhat shorter for younger students, but still giving them time to think and explain.

space, and cost have already been commended for assessing Claim 2. For Claim 4, the problem should be presented in a more open form, asking the student to identify the variables that need to be taken into account, and the information they will need to find. To make the distinction clear we illustrate this using the same problem situation: planning a table tennis tournament.

This is not a complete list (were such a thing possible); other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #4.

**Accessibility and Claim #4:** Many students with disabilities can analyze and create increasingly complex models of real world phenomena but have difficulty communicating their knowledge and skills in these areas. Successful adults with disabilities rely on alternative ways to express their knowledge and skills, including the use of assistive technology to construct shapes or develop explanations via speech to text. Others rely on calculators, physical objects, or tools for constructing shapes to work through their analysis and reasoning process.

For English learners, it will be important to recognize ELL students' linguistic background and level of proficiency in English in assigning tasks. It will also be important to include in the scoring process a discussion of ways to resolve issues concerning linguistic and cultural factors when interpreting responses.



**About the "Summative Assessment Targets" that follow…**

The following pages identify summative assessment targets that describe the evidence that will be used to support Claim #4. Summative assessment targets do not replace the Common Core standards; rather, they reference specific standards at each grade level that test developers will use to guide item and task development and collectively serve the purpose of providing a consistent sampling plan for assessment within and across grades.

The targets that are provided are for grades 4, 8, and 11, serving as elementary, middle, and high school examples of the targets that the Consortium will develop for grades 3-11. The summative assessment targets at each grade level represent the prioritized content for assessment. Suggested classroom-based interim and formative assessment targets are being developed, representing smaller learning chunks that teachers can use to monitor ongoing progress in the classroom of critical learning and/or content standards.

Each assessment target is accompanied by the related standard(s) in the CCSS from which it is drawn, and by the intended cognitive rigor/depth-of-knowledge (DOK) required by the assessment target. (The schema for DOK used here appears in Appendix B.)

## Grade 4 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #4- Data Analysis and Modeling

**Mathematics Claim #4**

**Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

**Content for this claim will be drawn from grade 4 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Operations & Algebraic Thinking**

1. **ANALYZING and MODELING OPERATIONS:** Analyze, interpret, and represent real-world scenarios or problems posed, using four operations and mathematical models, such as diagrams, tables, graphs, equations, rules, or visual models **Standards: 4.OA-1, 4.OA-2, 4.OA-4** (DOK 2, DOK 3)
2. **ANALYZING and MODELING PATTERNS:** Analyze, interpret, and represent real-world geometric and numeric patterns (including patterns with factors and multiples), using mathematical models, such as diagrams, tables, graphs, equations, rules, visual models **Standards: 4.OA-4, 4.OA-5** (DOK 2, DOK 3)

**Number & Operations Base Ten**

3. **MODELING PLACE VALUE :** Model place value concepts in real-world scenarios or problems posed, using mathematical models, such as equations, rectangular arrays, expanded form, and area models **Standards: 4.NBT-1, 4.NBT-2, 4.NBT-5, 4.NBT-6** (DOK 2)

**Number & Operations Fractions**

4. **MODELING FRACTIONS:** Explain and model how addition, subtraction, or multiplication of fractions changes a quantity, using mathematical models, such as equations and visual models **Standards: 4.NF-3; 4.NF-4** (DOK 2)

## Grade 8 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #4- Data Analysis and Modeling

**Mathematics Claim #4**

**Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

**Content for this claim will be drawn from grade 8 clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Expressions and Equations**

1. **ANALYZING and MODELING PROPORTIONAL RELATIONSHIPS:** Analyze, interpret, and represent real-world problems involving proportional relationships (e.g., distance-time), using mathematical models, such as diagrams, tables, graphs, equations, rules, or visual models to support arriving at the solution **Standards: 8. EE.5, 8.EE.6, 8.EE.7, 8.EE.8** (DOK 2)

**Functions**

2. **ANALYZING and MODELING FUNCTIONS:** Analyze a scenario or real-world problem (e.g., rate of change), using mathematical models, such as equations, graphs, tables, or verbal rules/descriptions to support arriving at the solution **Standards: 8.F.1, 8.F.2, 8.F-3, 8.F.4, 8.F-5** (DOK 2)

**Geometry**

3. **ANALYZING and MODELING GEOMETRIC CONCEPTS:** Analyze, interpret, and represent real-world problems involving geometric relationships and concepts, using mathematical models, such as composing-decomposing figures, using visual models and diagrams, using tables and graphs, or applying formulas and equations to support arriving at the solution **Standards: 8.G-2, 8.G.3, 8.G-4, 8.G-6. 8.G-8, 8.G-9** (DOK 2)

**Statistics and Probability**

4. **ANALYZING and MODELING BIVARIATE DATA:** Analyze and interpret data representing two real-world quantitative variables, using mathematical models, such as equations, diagrams, graphs, two-way tables, or visual models to support arriving at the solution **Standards: 8.SP-1, 8.SP-2, 8.SP-3, 8.SP-4** (DOK 2)

## Grade 11 SUMMATIVE ASSESSMENT TARGETS
## Providing Evidence Supporting Claim #4- Data Analysis and Modeling

### Mathematics Claim #4

**Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

**Content for this claim will be drawn from high school clusters consistent with a content prioritization as described in Appendix A, and may be assessed using a variety of items and tasks, either in stand-alone fashion or in combination in short and longer constructed responses or an extended performance task. Specific assessment tasks or scenarios may draw upon more than one content standard.**

**Number and Quantity**

1. **ANALYZING and MODELING QUANTITIES:** Analyze, interpret, and represent real-world quantities specific to the context in which they are used (e.g., acceleration, stopping distance; currency conversions, account balances; derived quantities such as person-hours and heating degree days; social science rates such as per-capita income; rates in everyday life such as points scored per game or batting averages) using mathematical models, such as diagrams, tables, graphs, equations, rules, flow charts, visual models **Standards: N-NQ-1, N-NQ-2, N-NQ-3** (DOK 2)

**Algebra**

2. **ANALYZE and MODEL using EXPRESSIONS, EQUATIONS & INEQUALITIES**: Analyze, interpret, and represent real-world problems (e.g., inequalities describing nutritional and cost constraints on combinations of different foods; expressions representing the return on money invested at an annualized percentage rate) involving the derivation, manipulation, or interpretation of equations, expressions, or inequalities using mathematical models to support solutions. **Standards: A.SSE-1, A.SSE-3, A.SSE-4, A.CED-2, A.CED-3, F.BF-1, F.BF-2** (DOK 2)

**Functions**

3. **ANALYZE and MODEL using GRAPHS:** Analyze, interpret, and represent real-world problems involving the use of verbal rules and graphs of functions to support solutions. **Standards: F.IF-5, F.IF-7, F.LE-1, F.LE-3, F.LE-5, F.TF-5** (DOK 2)

**Statistics and Probability**

4. **ANALYZE and MODEL DATA:** Decide whether a specified model (e.g., histogram, frequency table, graph, scatter plot, equation, rule, sample survey) is consistent with results/conclusions drawn from data-generating for a real-world scenario, including supporting estimations, margins or error, statistically significant outcomes, etc. **Standards: S-ID-1, S-ID-3, S-ID-4, S-ID-5, S-ID-6, S-ID-7 S-IC-2, S-IC- 4** (DOK 2)

## References

(Complete citations to be added for Round 2 version)

van Hiele, Pierre (1985) [1959], *The Child's Thought and Geometry*, Brooklyn, NY: City University of New York, pp. 243-252

**Appendix A: Content Analysis and Priorities**

This section presents grade-by-grade cluster-level priorities consistent the "Priorities in the *Common Core Standards'* Standards for Mathematical Content" recently developed by Zimba (2011, in press) as an extension and further elaboration of the Common Core State Standards, and their application to the development of curriculum priorities and assessment targets.

Zimba observes that "[n]ot everything in the Standards *should* have equal priority. For one thing, CCSSO and NGA charged the Working Group with basing the Standards on evidence. And in fact, the actual demands of college and careers elevate some material in the Standards to high importance for mastery, while making other material less important.

Moreover, at every level in mathematics there are intricate, challenging, and necessary things that serve as prerequisites for the next level's intricate, challenging, and necessary things. In order to keep as many students as possible on the path to readiness for college and careers, we need to give students enough time to succeed in these areas." (pp. 2-3) The content priorities proposed by Zimba, and duplicated in this Appendix take into account progressions of mathematical learning that are inherent in the CCSSM, and "have to be chosen with an eye to the arc of big ideas in the Standards."

The content priorities document is an analysis at the cluster level. Working at the cluster level helps to avoid obscuring the forest of big ideas behind the trees of specific standards (which are nevertheless individually important). Clusters serve as an appropriate grain size for following the contours of important progressions in the standards, such as the integration of place value understanding and the meanings and properties of operations that must take place as students develop computation strategies and algorithms for multi-digit numbers during grades K-6; or the appropriate development of functional thinking in middle school leading to the emergence of functions as a content domain in Grade 8.

Each cluster at each grade level, and for high school, has been assigned to one of three levels of priority – here represented as "***", "**", and "*". Zimba recommends that in an assessment context these levels should represent roughly a 70-20-10 breakdown, respectively, of time and focus on an assessment. The Consortium determination of sampling and selecting content for inclusion in its assessments will follow schema if not the same as that below, likely one of a similar nature.

## Mathematics | Grade 3

Prioritization of Mathematical Content Clusters

**Operations and Algebraic Thinking**

[***] Represent and solve problems involving multiplication and division.
[***] Understand properties of multiplication and the relationship between multiplication and division.
[***] Multiply and divide within 100.
[***] Solve problems involving the four operations, and identify and explain patterns in arithmetic.

**Number and Operations in Base Ten**

[**] Use place value understanding and properties of operations to perform multi-digit arithmetic.

**Number and Operations—Fractions**

[***] Develop understanding of fractions as numbers.

**Measurement and Data**

[***] Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
[*] Represent and interpret data.
[***] Geometric measurement: Understand concepts of area and relate area to multiplication and addition.
[**] Recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

**Geometry**

[**] Reason with shapes and their attributes.

**Summary of Mathematical Practices**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

# Mathematics | Grade 4

## Prioritization of Mathematical Content Clusters

**Operations and Algebraic Thinking**

[***] Use the four operations with whole numbers to solve problems.
[**] Gain familiarity with factors and multiples.
[*] Generate and analyze patterns.

**Number and Operations in Base Ten**

[***] Generalize place value understanding for multi-digit whole numbers.
[***] Use place value understanding and properties of operations to perform multi-digit arithmetic.

**Number and Operations—Fractions**

[***] Extend understanding of fraction equivalence and ordering.
[***] Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
[***] Understand decimal notation for fractions and compare decimal fractions.

**Measurement and Data**

[***] Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
[*] Represent and interpret data.
[**] Geometric measurement: Understand concepts of angle and measure angles.

**Geometry**

[**] Draw and identify lines and angles and classify shapes by properties of their lines and angles.

**Summary of Mathematical Practices**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

# Mathematics | Grade 5

## Prioritization of Mathematical Content Clusters

| | |
|---|---|
| **Operations and Algebraic Thinking**<br>[**] Write and interpret numerical expressions.<br>[*] Analyze patterns and relationships.<br><br>**Number and Operations in Base Ten**<br>[***] Understand the place value system.<br>[***] Perform operations with multi-digit whole numbers and with decimals to hundredths.<br><br>**Number and Operations—Fractions**<br>[***] Use equivalent fractions as a strategy to add and subtract fractions.<br>[***] Apply and extend previous understandings of multiplication and division to multiply and divide fractions.<br><br>**Measurement and Data**<br>[***] Convert like measurement units within a given measurement system.<br>[*] Represent and interpret data.<br>[***] Geometric measurement: Understand concepts of volume and relate volume to multiplication and to addition.<br><br>**Geometry**<br>[***] Graph points on the coordinate plane to solve real-world and mathematical problems.<br>[**] Classify two-dimensional figures into categories based on their properties. | **Summary of Mathematical Practices**<br><br>**1. Make sense of problems and persevere in solving them.**<br><br>**2. Reason abstractly and quantitatively.**<br><br>**3. Construct viable arguments and critique the reasoning of others.**<br><br>**4. Model with mathematics.**<br><br>**5. Use appropriate tools strategically.**<br><br>**6. Attend to precision.**<br><br>**7. Look for and make use of structure.**<br><br>**8. Look for and express regularity in repeated reasoning.** |

# Mathematics | Grade 6

## Prioritization of Mathematical Content Clusters

**Ratios and Proportional relationships**

[***] Understand ratio concepts and use ratio reasoning to solve problems.

**The Number System**

[***] Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

[**] Compute fluently with multi-digit numbers and find common factors and multiples.

[***] Apply and extend previous understandings of numbers to the system of rational numbers.

**Expressions and Equations**

[***] Apply and extend previous understandings of arithmetic to algebraic expressions.

[***] Reason about and solve one-variable equations and inequalities.

[***] Represent and analyze quantitative relationships between dependent and independent variables.

**Geometry**

[***] Solve real-world and mathematical problems involving area, surface area, and volume.

**Statistics and Probability**

[**] Develop understanding of statistical variability.

[**] Summarize and describe distributions.

**Summary of Mathematical Practices**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

## Mathematics | Grade 7

Prioritization of Mathematical Content Clusters

**Ratios and Proportional relationships**

[***] Analyze proportional relationships and use them to solve real-world and mathematical problems.

**The Number System**

[***] Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

**Expressions and Equations**

[***] Use properties of operations to generate equivalent expressions.

[***] Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

**Geometry**

[**] Draw, construct and describe geometrical figures and describe the relationships between them.

[***] Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

**Statistics and Probability**

[**] Use random sampling to draw inferences about a population.

[**] Draw informal comparative inferences about two populations.

[*] Investigate chance processes and develop, use, and evaluate probability models.

**Summary of Mathematical Practices**

**1. Make sense of problems and persevere in solving them.**

**2. Reason abstractly and quantitatively.**

**3. Construct viable arguments and critique the reasoning of others.**

**4. Model with mathematics.**

**5. Use appropriate tools strategically.**

**6. Attend to precision.**

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

## Mathematics | Grade 8

Prioritization of Mathematical Content Clusters

| The Number System / Expressions and equations / Functions / Geometry / Statistics and Probability | Summary of Mathematical Practices |
|---|---|
| **The Number System**<br>[**] Know that there are numbers that are not rational, and approximate them by rational numbers.<br><br>**Expressions and equations**<br>[***] Work with radicals and integer exponents.<br>[***] Understand the connections between proportional relationships, lines, and linear equations.<br>[***] Analyze and solve linear equations and pairs of simultaneous linear equations.<br><br>**Functions**<br>[***] Define, evaluate, and compare functions.<br>[***] Use functions to model relationships between quantities.<br><br>**Geometry**<br>[**] Understand congruence and similarity using physical models, transparencies, or geometry software.<br>[***] Understand and apply the Pythagorean Theorem.<br>[***] Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.<br><br>**Statistics and Probability**<br>[**] Investigate patterns of association in bivariate data. | **1. Make sense of problems and persevere in solving them.**<br><br>**2. Reason abstractly and quantitatively.**<br><br>**3. Construct viable arguments and critique the reasoning of others.**<br><br>**4. Model with mathematics.**<br><br>**5. Use appropriate tools strategically.**<br><br>**6. Attend to precision.**<br><br>**7. Look for and make use of structure.**<br><br>**8. Look for and express regularity in repeated reasoning.** |

**Content Priorities for High School**

The SMARTER Balanced Assessment Consortium will assess students' approach to, and attainment of college and career readiness. To a meaningful degree, what it takes to be ready for college and careers is an empirical question, and the Consortium's efforts will eventually be judged in part on the proven value of its assessments for informing postsecondary educational decisions. Assessments likely to pass this future test must account for facts about what college and careers actually require.

The high school table in this Appendix, below, highlights content clusters from the high school standards that figure most directly in attaining college and career readiness based on the analysis by Zimba (2011, in press). It should be noted that students who wish to pursue STEM careers, or who wish to do college-level work during high school (e.g., AP calculus, AP physics) must master all of the technical prerequisites for these subjects even though they may not be high priorities for college and career readiness generally.

As mentioned above, however, the consideration of content prioritization for high school needs to rely not only on the progression of important themes within the CCSSM, but also on the empirical evidence about which of the content areas in CCSSM are most important to post-secondary success. One significant source of such evidence is the views of college faculty themselves about what is required for success in college. Recent work in that area is newly available from David Conley and his colleagues at EPIC, who surveyed over 1,800 college instructors as to the applicability and importance of each of the CCSSM high school content standards.[19] The college instructors in the sample taught courses in many subjects, including English, Math, Science, Social Science, Business management, Computer technology, and Healthcare. Within mathematics, the courses included College Algebra, Statistics, and Calculus.

The Consortium will incorporate the findings of the recent EPIC study, and other relevant information, as its content prioritization and design work moves forward.

[19] Conley et al. citation to be added here.

# Mathematics | High School

## Prioritization of Mathematical Content Clusters

*(Clusters not appearing below are of third priority.)*

| Number and Quantity | Summary of Mathematical Practices |
| --- | --- |
| [***] Reason quantitatively and use units to solve problems.<br>[***] Extend the properties of exponents to rational exponents.<br>[**] Perform arithmetic operations with complex numbers.<br><br>**Algebra**<br>[***] Interpret the structure of expressions.<br>[***] Write expressions in equivalent forms to solve problems.<br>[***] Perform arithmetic operations on polynomials.<br>[***] Understand the relationship between zeros and factors of polynomials.<br>[***] Create equations that describe numbers or relationships.<br>[***] Understand solving equations as a process of reasoning and explain the reasoning.<br>[***] Solve equations and inequalities in one variable.<br>[**] Solve systems of equations.<br>[***] Represent and solve equations and inequalities graphically.<br><br>**Functions**<br>[**] Understand the concept of a function and use function notation.<br>[***] Interpret functions that arise in applications in terms of a context.<br>[***] Analyze functions using different representations.<br>[***] Build a function that models a relationship between two quantities.<br>[***] Construct and compare linear, quadratic, and exponential models and solve problems.<br>[***] Interpret expressions for functions in terms of the situation they model.<br><br>**…continued on next page** | **1. Make sense of problems and persevere in solving them.**<br><br>**2. Reason abstractly and quantitatively.**<br><br>**3. Construct viable arguments and critique the reasoning of others.**<br><br>**4. Model with mathematics.**<br><br>**5. Use appropriate tools strategically.**<br><br>**6. Attend to precision.**<br><br>**7. Look for and make use of structure.**<br><br>**8. Look for and express regularity in repeated reasoning.** |

# Mathematics | High School (continued)

## Prioritization of Mathematical Content Clusters

*(Clusters not appearing below are of third priority.)*

| Geometry | Summary of Mathematical Practices |
|---|---|
| [**] Experiment with transformations in the plane.<br>[**] Understand congruence in terms of rigid motions.<br>[**] Prove geometric theorems.<br>[**] Understand similarity in terms of similarity transformations.<br>[**] Prove theorems involving similarity.<br>[**] Define trigonometric ratios and solve problems involving right triangles.<br>[**] Find arc lengths and areas of sectors of circles.<br>[***] Translate between the geometric description and the equation for a conic section.<br>[***] Use coordinates to prove simple geometric theorems algebraically.<br>[***] Visualize relationships between two-dimensional and three-dimensional objects.<br>[***] Apply geometric concepts in modeling situations.<br><br>**Statistics and Probability**<br>[***] Summarize, represent, and interpret data on a single count or measurement variable.<br>[***] Summarize, represent, and interpret data on two categorical and quantitative variables.<br>[***] Interpret linear models.<br>[**] Understand and evaluate random processes underlying statistical experiments.<br>[***] Make inferences and justify conclusions from sample surveys, experiments and observational studies. | **1. Make sense of problems and persevere in solving them.**<br><br>**2. Reason abstractly and quantitatively.**<br><br>**3. Construct viable arguments and critique the reasoning of others.**<br><br>**4. Model with mathematics.**<br><br>**5. Use appropriate tools strategically.**<br><br>**6. Attend to precision.**<br><br>**7. Look for and make use of structure.**<br><br>**8. Look for and express regularity in repeated reasoning.** |

## Appendix B: Cognitive Rigor Matrix/Depth of Knowledge

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the “depth(s) of knowledge” that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom's (revised) Taxonomy of Educational Objectives and Webb’s Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks. (To download full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: [http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf](http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf))

### A “Snapshot” of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

| Depth of Thinking (Webb)<br>+ Type of Thinking (Revised Bloom) | DOK Level 1<br>Recall & Reproduction | DOK Level 2<br>Basic Skills & Concepts | DOK Level 3<br>Strategic Thinking & Reasoning | DOK Level 4<br>Extended Thinking |
|---|---|---|---|---|
| **Remember** | - Recall conversions, terms, facts | | | |
| **Understand** | -Evaluate an expression<br>-Locate points on a grid or number on number line<br>-Solve a one-step problem<br>-Represent math relationships in words, pictures, or symbols | - Specify, explain relationships<br>-Make basic inferences or logical predictions from data/observations<br>-Use models /diagrams to explain concepts<br>-Make and explain estimates | -Use concepts to solve non-routine problems<br>-Use supporting evidence to justify conjectures, generalize, or connect ideas<br>-Explain reasoning when more than one response is possible<br>-Explain phenomena in terms of concepts | -Relate mathematical concepts to other content areas, other domains<br>-Develop generalizations of the results obtained and the strategies used and apply them to new problem situations |
| **Apply** | -Follow simple procedures<br>-Calculate, measure, apply a rule (e.g., rounding)<br>-Apply algorithm or formula<br>-Solve linear equations<br>-Make conversions | -Select a procedure and perform it<br>-Solve routine problem applying multiple concepts or decision points<br>-Retrieve information to solve a problem<br>-Translate between representations | -Design investigation for a specific purpose or research question<br>- Use reasoning, planning, and supporting evidence<br>-Translate between problem & symbolic notation when not a direct translation | -Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results |
| **Analyze** | -Retrieve information from a table or graph to answer a question<br>-Identify a pattern/trend | -Categorize data, figures<br>-Organize, order data<br>-Select appropriate graph and organize & display data<br>-Interpret data from a simple graph<br>-Extend a pattern | -Compare information within or across data sets or texts<br>-Analyze and draw conclusions from data, citing evidence<br>-Generalize a pattern<br>-Interpret data from complex graph | -Analyze multiple sources of evidence or data sets |
| **Evaluate** | | | -Cite evidence and develop a logical argument<br>-Compare/contrast solution methods<br>-Verify reasonableness | -Apply understanding in a novel way, provide argument or justification for the new application |
| **Create** | - Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept | -Generate conjectures or hypotheses based on observations or prior knowledge and experience | -Develop an alternative solution<br>-Synthesize information within one data set | -Synthesize information across multiple sources or data sets<br>-Design a model to inform and solve a practical or abstract situation |